Published by Institute of Physics Publishing for SISSA/ISAS
Received: April 28, 2004
Accepted: May 11, 2004

Beauty is attractive: moduli trapping at enhanced symmetry points

Lev Kofman,[a] **Andrei Linde,**[b] **Xiao Liu,**[bc] **Alexander Maloney,**[bc] **Liam McAllister**[bc] **and Eva Silverstein**[bc]

[a] *CITA, University of Toronto Toronto, ON M5S 3H8, Canada*
[b] *Department of Physics, Stanford University Stanford, CA 94305, U.S.A.*
[c] *SLAC, Stanford University Stanford, CA 94309, U.S.A.*
E-mail: kofman@cita.utoronto.ca, alinde@stanford.edu, liuxiao@stanford.edu, maloney@itp.stanford.edu, lpm@itp.stanford.edu, evas@slac.stanford.edu

Abstract: We study quantum effects on moduli dynamics arising from the production of particles which are light at special points in moduli space. The resulting forces trap the moduli at these points, which often exhibit enhanced symmetry. Moduli trapping occurs in time-dependent quantum field theory, as well as in systems of moving D-branes, where it leads the branes to combine into stacks. Trapping also occurs in an expanding universe, though the range over which the moduli can roll is limited by Hubble friction. We observe that a scalar field trapped on a steep potential can induce a stage of acceleration of the universe, which we call trapped inflation. Moduli trapping ameliorates the cosmological moduli problem and may affect vacuum selection. In particular, rolling moduli are most powerfully attracted to the points with the largest number of light particles, which are often the points of greatest symmetry. Given suitable assumptions about the dynamics of the very early universe, this effect might help to explain why among the plethora of possible vacuum states of string theory, we appear to live in one with a large number of light particles and (spontaneously broken) symmetries. In other words, some of the surprising properties of our world might arise not through pure chance or miraculous cancellations, but through a natural selection mechanism during dynamical evolution.

Keywords: Superstring Vacua, Gauge Symmetry.

http://jhep.sissa.it/archive/papers/jhep052004030/jhep052004030.pdf

Contents

1. Introduction **2**
- 1.1 Moduli trapping near enhanced symmetry points 2
- 1.2 Relation to other works 4

2. Moduli trapping: basic mechanism **5**
- 2.1 Quantum production of χ particles 6
- 2.2 Backreaction on the motion of ϕ 7
- 2.3 The example of moving D-branes 9

3. Moduli trapping: detailed analysis **10**
- 3.1 Formal description of particle production near an ESP 10
- 3.2 Moduli trapping: numerical results 13
- 3.3 The special case of one-dimensional motion 13

4. Trapped moduli in an expanding universe **17**
- 4.1 Rapid trapping 17
- 4.2 Scanning range in an expanding universe 18
- 4.3 Trapping in an expanding universe 19
- 4.4 Efficiency of trapping 20

5. String theory effects **21**
- 5.1 Large χ mass 21
- 5.2 Large v and the Hagedorn density of states 22
- 5.3 Light field-theoretic strings 22

6. The vacuum selection problem **23**
- 6.1 Vacuum selection in Quantum Field Theory 23
- 6.2 Vacuum selection in supergravity and superstring cosmology 23
- 6.3 Properties of the resulting vacua 24

7. The moduli problem **25**

8. Trapped inflation and acceleration of the universe **26**

9. Conclusion **29**

A. Particle production due to motion on moduli space **29**

B. Annihilation of the χ particles **31**

C. Bending angle and energy loss during the first pass **33**

D. Classical trapping versus quantum trapping 36

1. Introduction

1.1 Moduli trapping near enhanced symmetry points

Supersymmetric string and field theories typically contain a number of light scalar fields, or moduli, which describe low-energy deformations of the system. If the kinetic energy of these fields is large compared to their potential energy then the classical dynamics of the moduli is described by geodesic motion on moduli space.

At certain special points (or subspaces) of moduli space, new degrees of freedom become light and can affect the dynamics of moduli in a significant way [1]–[5]. These extra species often contribute to an enhanced symmetry at the special point. We will refer to any points where new species become light as ESPs, which stands for extra species points, and also, when applicable, for enhanced symmetry points.

A canonical example is a system of two parallel D-branes. When the branes coincide, the two individual U(1) gauge symmetries are enhanced to a U(2) symmetry, as the strings that stretch between the branes become massless [6]. Similar points with new light species arise in many contexts; examples include the Seiberg-Witten massless monopole and dyon points in $\mathcal{N} = 2$ supersymmetric field theories [7], the conifold point [8] and ADE singularities in Calabi-Yau compactification [9], the self-dual radius of string compactifications on a torus, small instantons in heterotic string theory [10], and many other configurations with less symmetry.

Classically, there is no sense in which these ESPs are dynamically preferred over other metastable vacuum states of the system. We will argue that this changes once quantum effects are included. In particular, quantum particle production of the light fields alters the dynamics in such a way as to drive the moduli towards the ESPs and trap them there.

The basic mechanism of this trapping effect is quite simple. Consider a modulus ϕ moving through moduli space near an ESP associated to a new light field χ. For example, ϕ could be the separation between a pair of parallel D-branes, and χ a string stretching between the two branes — in this case the ESP $\phi = 0$ is the point where the branes coincide and χ becomes massless. As ϕ rolls through moduli space, the mass of χ changes; χ gets lighter as ϕ moves closer to the ESP and heavier as ϕ moves farther away. This changing mass leads to quantum production of χ particles; as ϕ moves past the ESP some of its kinetic energy will be dumped into χ particles. As ϕ rolls away from the ESP, more and more of its energy will be drained into the χ sector as the χ mass increases, until eventually ϕ stops rolling. At this point the moduli space approximation for ϕ has broken down, and all of the original kinetic energy contained in the coherent motion of ϕ has been transferred into χ particles, and ultimately into all of the fields interacting with χ (including decoherent quanta $\delta\phi$). As we will see in detail, the χ excitations generate a classical potential for ϕ which drives the modulus back toward the ESP and traps it there.[1]

In the example of the pair of moving D-branes, the consequences of this are simple: two parallel branes that are sent towards each other will collide and remain bound together. The original kinetic energy of the moving branes will be transferred into open string excitations on the branes and eventually into closed string radiation in the bulk.

In section 2 we will describe the general trapping mechanism and study its range of applicability using a few simple estimates. In section 3 we will write down the equations of motion governing trapping in more detail, and describe the numerical and analytic solutions of these equations in a variety of cases.

It is important to recognize that this trapping effect is in no way special to string theory. Flat space quantum field theory with a moduli space for ϕ and an ESP is an ideal setting for the trapping effect, and it is in this setting that we will perform the analysis of section 2 and section 3. In section 4 we will generalize this to incorporate the effects of cosmological expansion, and in section 5 we will discuss the possibility of significant effects from string theory. Having established the moduli trapping effect in a variety of contexts, we will then study its applications to problems in cosmology.

The most immediate application is to the problem of vacuum selection. As we will see in section 6, the trapping effect can provide a dynamical vacuum selection principle, reducing the problem to that of selecting one point within the class of ESPs. This represents significant progress, since the vast majority of metastable vacua are not ESPs. Trapping at ESPs may also help solve the cosmological moduli problem, as we will see in section 7. In particular, trapping strengthens the proposal of [11] by providing a dynamical mechanism which explains why moduli sit at points of enhanced symmetry.

Finally, as we will explain in section 8, the trapping of a scalar field with a potential can lead to a period of accelerated expansion, in a manner reminiscent of thermal inflation [12]. This effect, which we will call trapped inflation, can occur in a steeper potential than normally admits such behavior.

From a more general perspective, moduli trapping gives us insight into the celebrated question of why the world is so symmetric. The initial puzzle is that although highly symmetric theories are aesthetically appealing and theoretically tractable, they are also very special and hence, in an appropriate sense, rare. One expects that in a typical string theory vacuum, most symmetries will be strongly broken and most particles will have masses of order the string or Planck mass, just as in a typical vacuum one expects a large cosmological constant. Vacua with enhanced symmetry or light particles should comprise a minuscule subset of the space of all vacua.

Nevertheless, we observe traces of many symmetries in the properties of elementary particles, as spontaneously broken global and gauge invariances. Moreover, all known

[1] There are also corrections to the effective action for ϕ from loops of χ particles, including both kinetic corrections and a Coleman-Weinberg effective potential. Both effects will be subdominant in the weakly-coupled, supersymmetric, kinetic-energy dominated regimes we will consider.

particles are hierarchically light compared to the Planck mass. Given the expectation that a typical vacuum contains very few approximate symmetries and very few light particles, it is puzzling that we see such symmetries and such particles in our world.

For questions of this nature, moduli trapping may have considerable explanatory power. Specifically, the force pulling moduli toward a point of enhanced symmetry is proportional to the number of particles which become massless at this point, which is often associated with a high degree of symmetry. This means that the most attractive ESPs are typically the ones with the largest symmetry, and rolling moduli are most likely to be trapped at highly symmetric points, where many particles become massless or nearly massless. Moreover, the process of trapping can proceed sequentially: a modulus moving in a multi-dimensional moduli space can experience a sequence of trapping events, each of which increases the symmetry. These effects suggest that the symmetry and beauty we see in our world may have, at least in part, a simple dynamical explanation: beauty is attractive. We will discuss this possibility in section 6.

1.2 Relation to other works

Similar effects have been described in the literature. There has been much work on multi-scalar quantum field theory in the context of inflation, especially concerning preheating in interacting scalar field theories. Some of our results will be based on the theory of particle production and preheating developed in the series of papers [1, 2, 3], which explores many of the basic phenomena in scalar theories of the sort we will consider. Likewise, Chung et al. [4] have explored the effects of particle production on the inflaton trajectory and on the spectrum of density perturbations. Although we will derive what we need here in a self-contained way, many of the technical results in this paper overlap with those works, as well as with standard results on particle production in time-dependent systems as summarized in e.g. [13]. Although we will not study the case in which χ goes tachyonic for some range of ϕ, our results may nevertheless have application to models of hybrid inflation [14, 15], including models based on rapidly-oscillating interacting scalars [16, 17, 18].

In strong 't Hooft coupling regions of moduli spaces which are accessible through the AdS/CFT correspondence, virtual effects from the large numbers of light species dramatically slow down the motion of ϕ as it approaches an ESP, with the result that the modulus gets trapped there [5]. This also provides a mechanism for slow roll inflation without very flat potentials. In the present work, which applies at weak 't Hooft coupling, it is quantum production of *on-shell* light particles which leads to trapping on moduli space.

Other works in the context of string theory have explored the localization of moduli at ESPs. The authors of [19, 20] studied the evolution of a supersymmetric version of the $\phi - \chi$ system arising near a flop transition using an effective supergravity action. They showed that, given nonvanishing initial vevs for both ϕ and χ, the fields will settle at the ESP even if one formally turns off particle production effects. Our proposal, by contrast, is to take into account on-shell quantum effects which dynamically generate a nonzero $\langle\chi^2\rangle$. In works such as [21] attention was focused on the boundaries of moduli space, while here we focus on ESPs in the interior of moduli space. In [22], production of light strings was studied in the context of D0-brane quantum mechanics; as we explain in section 2.3, this has some similarities, but important differences, with our case of space-filling branes. Scattering of Dp-branes was also studied in [23].

Dine has suggested that enhanced symmetry points may provide a solution to the moduli problem, as moduli which begin at an enhanced symmetry minimum of the quantum effective potential can consistently remain there both during and immediately after inflation [11]. One would still like to explain why the moduli began at such a point. As we discuss in section 7, our trapping mechanism provides a natural explanation for this initial configuration.

Horne and Moore [24] have argued that the classical motion on certain moduli spaces is ergodic, provided that the potential energy is negligible. This means that all configurations are sampled given a sufficiently long time, and in particular a given modulus will eventually approach an ESP. We will argue that quantum corrections to the classical trajectory are significant, and indeed lead to trapping, whenever the classical trajectory comes close to an ESP. Combining these two observations, we expect that in the full, quantum-corrected system the moduli are stuck near an ESP at late times. This means that the quantum-corrected evolution is not fully ergodic: the dynamics of [24] (see also [25]) implies that the modulus will eventually approach an ESP, at which point quantum effects will trap it there, preventing the system from sampling any further regions of moduli space.

2. Moduli trapping: basic mechanism

We will now describe the mechanism of moduli trapping in more detail. Our discussion in this section will be based on simple estimates of particle production and the consequent backreaction, generalizing the results of [1, 2, 3] to the case of a complex field. A more complete analysis, along with numerical results, will be presented in section 3.

We will consider the specific model

$$\mathcal{L} = \frac{1}{2}\partial_\mu \phi \partial^\mu \bar{\phi} + \frac{1}{2}\partial_\mu \chi \partial^\mu \chi - \frac{g^2}{2}|\phi|^2\chi^2 \tag{2.1}$$

where a complex modulus $\phi = \phi_1 + i\phi_2$ interacts with a real scalar field χ. We are restricting ourselves to the case of a flat moduli space which has a single ESP at $\phi = 0$, where χ becomes massless, and a particularly simple form for the χ interaction. This simple case illustrates the basic physics and can be generalized as necessary, for example to include supersymmetry.

We will consider the case where ϕ approaches the origin with some impact parameter μ, following a classical trajectory of the form

$$\phi(t) = i\mu + vt\,. \tag{2.2}$$

Classically, if χ vanishes then (2.2) is an exact solution to the equations of motion, and the presence of the ESP will not affect the motion of ϕ.

Quantum effects will alter this picture considerably, because the trajectory (2.2) will lead to the production of χ particles, as we discuss in section 2.1. The backreaction of these particles on the motion of ϕ will then lead to trapping, as we will see in section 2.2. In section 2.3 we will illustrate this effect with the example of colliding D-branes.

2.1 Quantum production of χ particles

Let us first study the creation of χ particles without considering how they may backreact to alter the motion of ϕ. In this approximation we may substitute (2.2) into the action (2.1) to get a free quantum field theory for χ with a time-varying mass

$$m_\chi^2(t) = g^2|\phi(t)|^2 . \tag{2.3}$$

This time dependence leads to particle production.

Consider a mode of the χ field with spatial momentum k, whose frequency

$$\omega(t) = \sqrt{k^2 + g^2|\phi(t)|^2} \tag{2.4}$$

varies in time. This mode becomes excited when the non-adiabaticity parameter $\dot{\omega}/\omega^2$ becomes at least of order one. This parameter vanishes as $t \to \pm\infty$, indicating that particle creation takes place only while ϕ is near the ESP. It is straightforward to see that, for the trajectory (2.2), $\dot{\omega}/\omega^2$ can be large only in the small interval $|\phi| \lesssim \Delta\phi$ near the ESP, where

$$\Delta\phi = \sqrt{\frac{v}{g}}, \tag{2.5}$$

and only for momenta

$$\frac{k^2 + g^2\mu^2}{gv} \lesssim 1 . \tag{2.6}$$

When the quantity on the left hand side is small, particle creation effects are very strong. They are strongest if the modulus passes sufficiently close to the ESP, i.e. if

$$\mu \lesssim \sqrt{\frac{v}{g}} . \tag{2.7}$$

In this case χ modes whose momenta k fall in the range (2.6) will be excited.[2] Qualitatively, we expect that the occupation numbers n_k of such modes will vary from zero (no real particles) for modes with vanishing non-adiabaticity to of order unity for modes with very large non-adiabaticity. The full computation of n_k given in appendix A yields

$$n_k = \exp\left(-\pi\frac{k^2 + g^2\mu^2}{gv}\right), \tag{2.9}$$

[2]This may be checked as follows. We have argued that unsuppressed particle production occurs only when the modulus is sufficiently close to the ESP, $|\phi| \lesssim \sqrt{v/g}$. The modulus remains within this window for a time

$$\Delta t \sim \frac{\sqrt{v/g}}{v} \sim (gv)^{-1/2} . \tag{2.8}$$

The uncertainty principle implies in this case that the created particles will have typical energy $E \sim (\Delta t)^{-1}$ and thus momenta $k \sim (gv - g^2\mu^2)^{1/2}$. This agrees with the estimate (2.6).

which agrees with this qualitative expectation. Note that even when (2.7) is not satisfied, there is generically a nonvanishing, though exponentially suppressed, number density of created particles; even in this case we will find a nontrivial trapping effect.

Before discussing the backreaction due to the production of χ particles, it is crucial to control other effects from the χ field. In particular, there is another important quantum effect which arises in motion toward the origin: loops of light χ particles give corrections to the effective action. These include both kinetic corrections and the Coleman-Weinberg potential energy. The latter we will subtract by hand, as we will explain in section 3.1. This gives a good approximation to the dynamics in any situation where kinetic energy dominates.

The kinetic corrections are organized in an expansion in v^2/ϕ^4 [5]. The parameters controlling both remaining effects — the nonadiabaticity controlling particle production and the kinetic factor v^2/ϕ^4 controlling light virtual χ particles — diverge as we approach the origin. However, at weak coupling, the nonadiabaticity parameter is parametrically enhanced relative to the kinetic corrections, i.e. $v^2/g^2\phi^4 \gg v^2/\phi^4$, so we can sensibly focus on the effects of particle production. More specifically, we can ensure that the kinetic corrections are insignificant by including a sufficiently large impact parameter μ.

We will also analyze the case of small μ, including $\mu = 0$. This relies on the plausible assumption that the effects of the kinetic corrections remain subdominant as we approach very close to the origin, and that in particular in our weak coupling case they do not by themselves stop ϕ from progressing through the origin. It would be interesting to develop theoretical tools to analyze this issue more directly and check this hypothesis.

2.2 Backreaction on the motion of ϕ

One might expect *a priori* that any description of the motion of ϕ which fully incorporates backreaction from particle production would be immensely complicated. Fortunately, this turns out not to be the case, and a simple description is possible. The key simplification is that creation of χ particles happens primarily in a small vicinity of the ESP $\phi = 0$, so one can treat this as an instant event of particle production. These particles induce a very simple linear, confining potential acting on ϕ, $V \sim |\phi|$. The motion of ϕ in this potential between successive events of particle production can be described rather simply.

Let us now explore this in more detail. We have seen that as ϕ moves in moduli space, some of its energy will be transferred into excitations of χ. This leads to a quantum vacuum expectation value $\langle\chi^2\rangle \neq 0$. As ϕ rolls away from the ESP, the mass of the created χ particles increases, further increasing the energy contained in the χ sector. At this point the backreaction of the χ field on the dynamics of ϕ becomes important, and the moduli space approximation breaks down.

We will concentrate on the backreaction of the created particles on the motion of the field ϕ far away from the small region of non-adiabaticity, i.e. for $\phi \gg \Delta\phi \sim \sqrt{v/g}$. At this stage the typical momenta are such that the χ particles are nonrelativistic, $k \lesssim \sqrt{gv} \ll g|\phi|$. Therefore the total energy density of the gas of χ particles is easily seen to be

$$\rho_\chi(\phi) = \int \frac{d^3k}{(2\pi)^3} n_k \sqrt{k^2 + g^2|\phi(t)|^2} \approx g|\phi(t)|n_\chi\,, \tag{2.10}$$

where n_χ is the number density of χ particles,

$$n_\chi = \int \frac{d^3k}{(2\pi)^3} n_k = \frac{(gv)^{3/2}}{(2\pi)^3} e^{-\pi g\mu^2/v} \tag{2.11}$$

As ϕ continues to move away from the ESP $\phi = 0$, the number density of χ particles remains constant, as particles are produced only in the vicinity of $\phi = 0$. However, the energy density of the χ particles grows as $g|\phi(t)|n_\chi$. This leads to an attractive force of magnitude gn_χ, which always points towards the ESP $\phi = 0$.

This force of attraction slows down the motion of ϕ, and eventually turns ϕ back toward the ESP. This reversal occurs in the vicinity of the point ϕ_* at which the initial kinetic energy density $\frac{1}{2}\dot{\phi}^2 \equiv \frac{1}{2}v^2$ matches the energy density ρ_χ contained in χ particles. We find

$$\phi_* = \frac{4\pi^3}{g^{5/2}} v^{1/2} e^{\pi g\mu^2/v} \,. \tag{2.12}$$

Observe that for $g \ll 1$ the trapping length on the first pass is always much greater than the impact parameter μ, which means that the motion of the moduli after the first impact is effectively one-dimensional.

After changing direction at ϕ_*, ϕ falls back toward the origin. On this second pass by the ESP, more χ particles are produced, leading to a stronger attractive force. This process repeats itself, leading ultimately to a trapped orbit of ϕ about the ESP, in a trajectory determined by the effective potential and consistent with angular momentum conservation on moduli space.

We conclude that, in this simplified setup, a scalar field which rolls past an ESP will oscillate about the ESP with an initial amplitude given by (2.12).

In fact, in many cases the amplitude of these oscillations will rapidly decrease due to the effect of parametric resonance, similar to the effects studied in the theory of preheating [1], and the field ϕ will fall swiftly towards the ESP. This important result will be described in more detail in section 3.3.

So far we have not incorporated the effects of scattering and decay of the χ particles. These could weaken the trapping potential (2.10) by reducing the number of χ particles. Specifically, the energy density ρ_χ contained in a fixed number of χ particles (2.10) grows at late times, since the χ mass increases as ϕ rolls away from the ESP. However, if the number density of χ particles decreases due to annihilation or decay into lighter modes, this mass amplification effect is lost. It is therefore important to determine the rate of decay and annihilation of the χ particles.

In appendix B we address these issues and demonstrate that the trapping effect is robust for certain parameter ranges, provided that the light states are relatively stable. This stability can easily be arranged in supersymmetric models, and in fact occurs automatically in certain D-brane systems.

Rescattering effects, in contrast, may actually strengthen the trapping effect. Once χ particles have been created, they will scatter off of the homogeneous ϕ condensate, causing it to gradually decay into inhomogeneous, decoherent ϕ excitations [1, 26, 27]. However, we will not consider this potentially beneficial effect here.

JHEP05(2004)030

2.3 The example of moving D-branes

Before proceeding, it may be illustrative to discuss these results in terms of a simple, mechanical example — a moving pair of D-branes. The moduli space of a system of two D-branes is the space of brane positions. In terms of the brane worldvolume fields the separation between the two branes can be regarded as a Higgs field ϕ. The off-diagonal components of the U(2) gauge field are the W bosons. At the ESP of this system, $\phi = 0$, the W bosons are massless. Away from $\phi = 0$ the W bosons acquire a mass by the Higgs mechanism, breaking the symmetry group from U(2) down to U(1) $\times$ U(1). If we identify χ with the W field[3] and $g^2 \sim g_{\mathrm{YM}}^2 \sim g_s$ with the string coupling, then we find that the brane worldvolume theory contains a term like (2.1). We therefore expect this system to exhibit moduli trapping.

The trapping effect is a quantum correction to the motion of D-branes. As the D-branes approach each other, the open strings stretched between them become excited. When the D-branes pass by each other and begin moving apart the stretched open strings become massive and pull the D-branes back together. We depict this in figure 1.

This effect can be a significant correction to the dynamics of any system with a number of mobile, mutually BPS D-branes. Consider, for example, N D3-branes which fill spacetime and are transverse to a compact six-manifold M. Let us take these branes to begin with small, random, classical velocities in M. The classical dynamics of this system is similar to that of a nonrelativistic, noninteracting, classical gas. When we include quantum production of light strings, the branes begin to trap each other, pairwise or in small groups, then gradually agglomerate until only a few massive clumps of many branes remain.

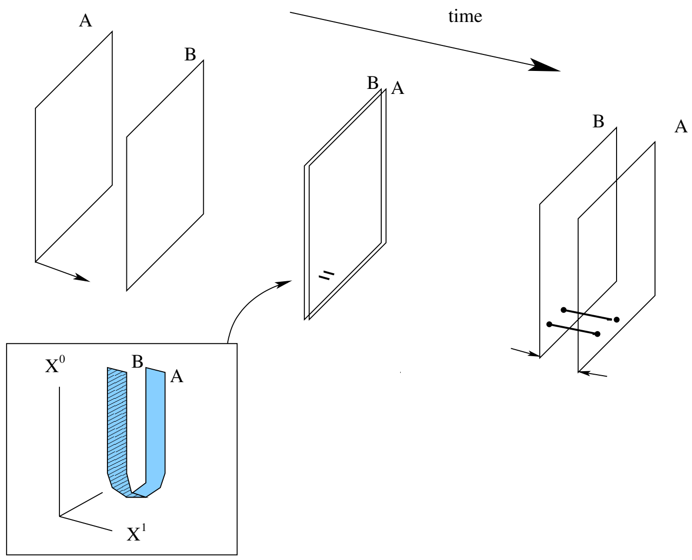

Figure 1: This figure illustrates the creation of open strings as two D-branes pass near each other. The left corner shows the target space picture of the creation of the open strings.

[3]For simplicity we ignore the superpartner of the χ boson.

One interesting consequence is that such a system will tend to exhibit enhanced gauge symmetry, with gauge group $\mathrm{U}(N)$ if the final state consists of a single clump. (Hubble friction may bring the branes to rest before the aggregation is complete, in which case the gauge group will be a product of smaller factors; we will address related issues in section 4.1.) Another important effect of massive clumps is their gravitational backreaction: a large cluster of D-branes will produce a warped throat region in M, which may be of phenomenological interest [28].

There are additional corrections to the classical moduli space approximation of the D-brane motion which come from velocity-dependent forces. These correspond in the D-brane worldvolume field theory to higher-derivative corrections generated by virtual effects. When this field theory is at weak 't Hooft coupling, open string production is the dominant effect as one approaches an ESP. However, sufficiently large clusters of branes will be described by gauge theories at strong 't Hooft coupling, where the dynamics of additional probe branes is governed instead by the analysis of [5].[4]

A similar interaction was studied in the context of the scattering of D0-branes in [22]. There is a crucial difference between that system and the case of interest here, in which the branes are extended along $3+1$ dimensions. In the D0-brane problem, there is a nontrivial probability for the D0-branes to pass by each other without getting trapped: because the D0-brane is pointlike, there is some probability for no open strings between them to be created or for those created to annihilate rapidly. This is the leading contribution to the S-matrix. In our case, there is always a nonzero number *density* of particles created. As we argue in appendix B, for certain ranges of parameters these particles do not annihilate rapidly enough to prevent trapping.

3. Moduli trapping: detailed analysis

In the previous section we gave an intuitive explanation of the trapping effect, which we will now describe in more detail. In section 3.1 we will present the equations of motion which govern the trajectory of the modulus ϕ, including the backreaction due to production of light particles. These equations are difficult to solve exactly, so in section 3.2 we will integrate the system numerically. In section 3.3 we focus on the special case $\mu = 0$, where the modulus rolls directly through the ESP. In this case analytic techniques are available, and as we will see the trapping effect is considerably stronger than in the $\mu \neq 0$ case. Even for $\mu \neq 0$ some aspects can be derived analytically in a perturbative expansion. This last case is somewhat technical, so we refer the reader to appendix C for details.

3.1 Formal description of particle production near an ESP

The full equations of motion are found by coupling the classical motion of ϕ to the time-dependent χ quantum field theory defined by (2.1).[5]

[4] A further correction to our dynamics could arise if, as we will discuss in section 5, the branes keep moving until the system is beyond the range of effective field theory.

[5] We remain in flat space quantum field theory, reserving gravitational effects for section 4.

JHEP05(2004)030

In general, the presence of an ESP will alter the moduli dynamics in two ways. First, any χ excitations produced by the mechanism described above will backreact on the classical evolution of ϕ. In particular, as we saw in (2.10), a non-zero expectation value $\langle\chi^2\rangle \neq 0$ arising from particle production effectively acts like a linear potential for ϕ and drives the moduli towards the origin. This is the effect we wish to describe. Second, virtual χ particles generate quadratic and higher-derivative contributions to the effective action as well as an effective potential for a spacetime-homogeneous ϕ.

As we discussed in section 2.1, we can neglect the kinetic corrections in our weakly-coupled situation. The interaction in (2.1) also induces important radiative corrections to the effective potential. Specifically, it leads to a Coleman-Weinberg effective potential and three UV-divergent terms:

$$V_{\text{eff}}(\phi) = \Lambda_{\text{eff}} + g^2 m_{\text{eff}}^2 \phi^2 + g^4 \lambda_{\text{eff}} \phi^4 \,. \tag{3.1}$$

These UV divergences could be subtracted by hand using appropriate counterterms. In a supersymmetric system these divergences are absent.

In order to isolate the effects of particle production at the order we are working, we will subtract by hand the entire Coleman-Weinberg effective potential for ϕ that is generated by one loop of χ particles. This mimics the effect of including extended supersymmetry, which is a toy case of interest in string theory and supergravity. For the more realistic $\mathcal{N}=1$ supersymmetry in four dimensions, radiative corrections do generically generate a nontrivial potential energy. Nevertheless, particle production effects can still dominate the virtual corrections to the potential after spontaneous supersymmetry breaking. The reason is that bosons and fermions contribute with opposite signs in loops, but on-shell bosons and fermions, such as those produced by the changing mass of χ, contribute with the same sign to backreaction on ϕ.

To describe the production of χ particles, we first expand the quantum field χ in terms of Fock space operators as

$$\chi = \sum_k a_k \chi_k + a_k^\dagger \chi_k^* \tag{3.2}$$

where the χ_k are a complete set of positive-frequency solutions to the Klein-Gordon equation with mass

$$m_\chi^2(t) = g^2 |\phi(t)|^2 \,. \tag{3.3}$$

Expanding in plane waves

$$\chi_k = u_k(t) e^{ik\cdot x} \tag{3.4}$$

the equation of motion is

$$\left(\partial_t^2 + k^2 + g^2|\phi(t)|^2\right) u_k = 0 \,. \tag{3.5}$$

The modes (3.4) are normalized with respect to the Klein-Gordon inner product, which fixes

$$u_k^* \dot{u}_k - \dot{u}_k^* u_k = -i \,. \tag{3.6}$$

The wave equation (3.5) has two linearly-independent solutions for each k, so in general there will be many inequivalent choices of positive-frequency modes χ_k. Each such choice

JHEP05(2004)030

of mode decomposition defines a set of Fock space operators via (3.2), which in turn define a vacuum state of the theory. The wave equation depends explicitly on time, so there is no canonical choice of Poincaré invariant vacuum. Instead, there is a large family of inequivalent vacua for χ.

We can choose a set of positive frequency modes $u_k^{\rm in}$ that take a particularly simple form in the far past,

$$u_k^{\rm in} \to \frac{1}{\sqrt{2\sqrt{k^2+g^2|\phi|^2}}} e^{-i\int^t \sqrt{k^2+g^2|\phi(t')|^2}dt'} \qquad \text{as } t \to -\infty\,. \tag{3.7}$$

This choice of mode decomposition defines a vacuum state $|in\rangle$. In the far past the phases of the solutions (3.7) are monotone decreasing with t, indicating that the state $|in\rangle$ has no particles in the far past. This state, known as the adiabatic vacuum, evolves into a highly excited state as the modulus ϕ rolls past the ESP.

We can now write down the classical equation of motion for ϕ including the effects of χ production. Including a subtraction δ_M, to be determined shortly, it is

$$\Big(\partial^2 + g^2(\langle\chi^2\rangle - \delta_M)\Big)\phi = 0. \tag{3.8}$$

The expectation value $\langle\chi^2\rangle$ depends on time and is calculated in the adiabatic vacuum $|in\rangle$. At time t

$$\langle {\rm in}|\chi^2(t)|{\rm in}\rangle = \int \frac{d^3k}{(2\pi)^3}|u_k^{\rm in}(t)|^2\,. \tag{3.9}$$

where the $u_k^{\rm in}$ are determined by the boundary condition (3.7) in the far past.

In order to subtract the Coleman-Weinberg potential, we must remove the contribution to $\langle\chi^2\rangle$ coming from one loop of χ particles, replacing the χ mass-squared with $g^2|\phi(t)|^2$. That is, the subtraction δ_M can be written as

$$\delta_M \equiv \int \frac{d^3k}{(2\pi)^3}\frac{1}{2\sqrt{k^2+g^2|\phi|^2}}\,. \tag{3.10}$$

With this form it is straightforward to see that when the impact parameter is very large, $(\langle\chi^2\rangle - \delta_M)$ is negligible and ϕ follows its original trajectory (2.2).

To summarize, the effects of quantum production of χ particles on the classical motion of the modulus ϕ are governed by:

$$\begin{aligned}\Big(\partial^2 + g^2(\langle\chi^2\rangle - \delta_M)\Big)\phi &= 0\\ \Big(\partial_t^2 + k^2 + g^2|\phi(t)|^2\Big)u_k^{\rm in} &= 0\\ \langle\chi^2(t)\rangle &= \int \frac{d^3k}{(2\pi)^3}|u_k^{\rm in}(t)|^2\,.\end{aligned} \tag{3.11}$$

The above equations of motion can be reformulated in terms of the energy transferred between the two systems. In particular, it is straightforward to show that the coupled equations (3.11) are equivalent to the statement

$$\frac{d}{dt}H_\phi = -\frac{d}{dt}\langle{\rm in}|H_\chi|{\rm in}\rangle\,. \tag{3.12}$$

The left-hand side of (3.12) involves the classical energy of the rolling $\phi(t)$ fields, whereas the right hand side is an expectation value of the time-dependent χ hamiltonian calculated in quantum field theory. This is the more precise form of energy conservation which applies to our rough estimate in section 2.2.

Furthermore, the angular momentum on moduli space is conserved, since the action (2.1) is invariant under phase rotations $\phi \to \phi e^{i\theta}$. In the present case (2.1), the χ particles do not carry angular momentum, so the orbit of ϕ around the ESP will have fixed angular momentum. The result is an angular momentum barrier which keeps the modulus at a finite distance from the ESP.

More complicated scenarios allow for the exchange of angular momentum between ϕ and χ. This includes the case of colliding D-branes, where the strings stretching between the two D-branes can carry angular momentum. Moreover, as we will see in section 4, the situation changes once gravitational effects are included, as angular momentum is redshifted away by cosmological expansion. This leads to scenarios where the moduli are trapped exactly at the ESP, rather than orbiting around it at some finite distance.

3.2 Moduli trapping: numerical results

The coupled set of integral and differential equations (3.11) governing the trapping trajectory is hard to solve in general. Some analytic results can be obtained through an expansion in the non-adiabaticity parameter $\dot{\omega}/\omega^2$, combined with a systematic iteration procedure. Specifically, to the extent that this non-adiabaticity parameter is small enough that the correction to the motion of the moduli field only shows up at exponentially late times, we can calculate the bending angle and the energy loss during its first pass. This is given in appendix C.

As time goes on, the mass amplification of the χ particles makes higher order terms as well as non-perturbative terms in the adiabatic expansion crucial for the motion of the moduli. This makes it very hard to proceed analytically to obtain the detailed evolution of the system.

We have numerically integrated the coupled equations (3.11) in Mathematica, using a discrete sum to approximate the momentum integral k, and implementing the subtraction of the Coleman-Weinberg potential described above.

In figure 2 we plot a trajectory for the case $\mu > 0$, where ϕ becomes trapped in a spiral orbit around the ESP. The radius of the orbit varies with the parameters, but the qualitative features shown are typical.

In figure 3 we plot the trajectory of a modulus which is aimed to pass directly through an ESP, with vanishing impact parameter. In this case the motion becomes effectively one-dimensional, and the field moves directly through the ESP $\phi = 0$. The trapping effect in this case is especially strong, and can be understood analytically to come from resonant production of χ particles, as we will now explain.

3.3 The special case of one-dimensional motion

In this section we will concentrate on the interesting and important special case of one-dimensional motion, i.e. vanishing impact parameter μ. Perhaps surprisingly, this is a

JHEP05(2004)030

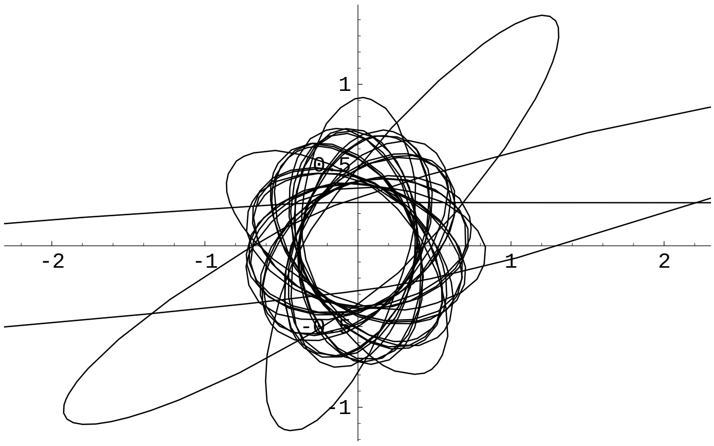

Figure 2: This figure shows the evolution, in the complex ϕ plane, of a system with parameters $g^2 = 20, \mu = 0.3, v = 1$. The field rolls in from the right and gets trapped into the precessing orbit exhibited in the plot. The orbit is initially an elongated ellipse, but gradually becomes more circular. In an expanding universe, the field would lose its angular momentum, so that the radius of the circle would eventually shrink to zero.

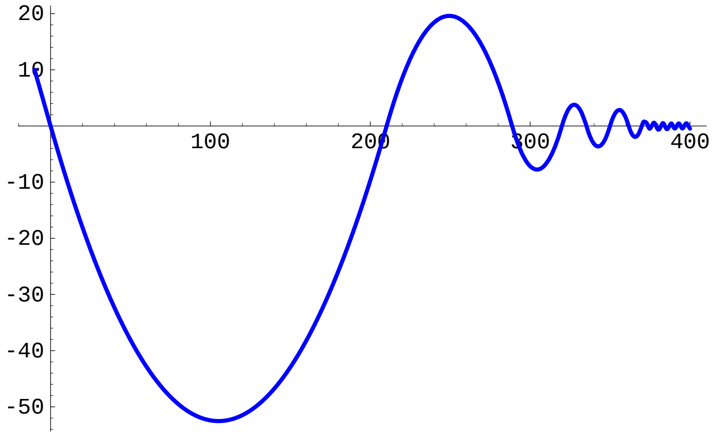

Figure 3: This shows one-dimensional trapping, in which ϕ passes directly through the ESP $\phi = 0$. The vertical axis is the real part of ϕ, and the horizontal axis is time. The amplitude of the oscillations decreases exponentially as a result of parametric resonance, as we explain in section 3.3.

good approximation to the general case. Indeed, the results of section 2 demonstrate that trapping becomes exponentially suppressed when the impact parameter μ (the imaginary part of the moduli field) becomes greater than $\sqrt{v/\pi g}$. On the other hand, for $\mu \ll \sqrt{v/\pi g}$ the motion of the field ϕ stops at $\phi_* \sim 4\pi^3 v^{1/2}/g^{5/2}$. The ratio of ϕ_* to μ in the regime where trapping is efficient (i.e. for $\mu < \sqrt{v/\pi g}$) is therefore

$$\frac{\phi_*}{\mu} > \frac{4\pi^{7/2}}{g^2}\,. \tag{3.13}$$

Thus, in the case of efficient trapping and weak coupling, the ellipticity of the moduli orbit is very high, so that the motion is effectively one-dimensional.

In the case $\mu = 0$ the number density of χ particles created when the field ϕ passes the ESP is

$$n_\chi = \frac{(gv)^{\frac{3}{2}}}{(2\pi)^3}\,. \tag{3.14}$$

At $|\phi| \gg \sqrt{v/g}$, when the χ particles are nonrelativistic, the mass of each particle is equal to $g|\phi|$, and their energy density is given by [2]

$$\rho_\chi(\phi) = g n_\chi |\phi| = \frac{g^{\frac{5}{2}} v^{3/2}}{(2\pi)^3}\,|\phi| \tag{3.15}$$

We have written $|\phi|$ because this energy does not depend on the sign of the field ϕ. This will be very important for us in what follows.

One should note that, strictly speaking, the χ particles have some kinetic energy even at $\phi = 0$, but for $g \ll 1$ this energy is much smaller than the kinetic energy of ϕ [2]:

$$\rho_\chi(\phi = 0) \sim \frac{g^2}{4\pi^{7/2}}\,\frac{v^2}{2} = \frac{g^2}{4\pi^{7/2}}\,\rho_\phi^{\rm kin}\,. \tag{3.16}$$

This means that the energy of ϕ decreases only slightly when it passes through the ESP $\phi = 0$. Although the initial energy in χ particles is small, this energy increases with $|\phi|$, $\rho_\chi \sim g n_\chi |\phi|$, and creates an effective potential for ϕ. The equation of motion for ϕ in this potential is [1]:

$$\ddot\phi + g n_\chi \frac{\phi}{|\phi|} = 0\,. \tag{3.17}$$

The last term means that ϕ is attracted to the ESP $\phi = 0$ with a constant force proportional to n_χ.

At some location ϕ_1^* the χ energy density ρ_χ equals the initial kinetic energy density $\frac{1}{2}\dot\phi^2 \equiv \frac{1}{2}v^2$; at this point ϕ stops and then falls back toward $\phi = 0$.

On this second pass by the origin, the energy density of the χ particles again becomes much smaller than the kinetic energy of ϕ. Energy conservation implies that ϕ will pass the point $\phi = 0$ at almost exactly the initial velocity v. Since the conditions are almost the same as on the first pass, new χ particles will be created, i.e. n_χ will increase. The field ϕ will continue moving for a while, stop at some point ϕ_2^*, and then fall back once more to the ESP, creating more particles. Because each new collection of particles is created in the presence of previous generations of particles, the process occurs in the regime of parametric resonance, as in the theory of preheating.

A detailed theory of this process was considered in [1]; see in particular eqs. (59), (60). By translating the problem into a one-dimensional quantum mechanics system (as in appendix A) with a particle scattering repeatedly across an inverted harmonic potential, [1] calculated the multiplicative increase of the Bogoliubov coefficients during each pass in terms of the reflection and transition amplitudes. In application to our problem, the equations describing the occupation numbers of χ particles with momentum k produced when the field passes through the ESP $j+1$ times look as follows:

$$n_k^{j+1} = n_k^j \exp(2\pi\mu_k^j)\,, \tag{3.18}$$

where

$$\mu_k^j = \frac{1}{2\pi} \ln \left(1 + 2e^{-\pi\xi^2} - 2\sin\theta^j \, e^{-\frac{\pi}{2}\xi^2} \, \sqrt{1+e^{-\pi\xi^2}} \right). \tag{3.19}$$

Here $\xi^2 = k^2/gv$ and θ^j is a relative phase variable which takes values from 0 to 2π. In a cyclic particle creation process in which the parameters of the system change considerably during each oscillation (which is our case, as will become clear shortly), the phases θ^j change almost randomly. As a result, the coefficient μ_j for small k takes different values, from 0.28 to -0.28, but for 3/4 of all values of the angle θ^j the coefficient μ_j is positive. The average value of μ_j is approximately equal to 0.15. This means that, on average, the number density of χ particles grows by approximately a factor of two or three each time that ϕ passes through the ESP $\phi = 0$.

But this means that with each pass, the coefficient n_χ in (3.15) grows by a factor of two or three. It follows that the effective potential becomes two to three times more steep with each pass. Correspondingly, the maximal deviation $|\phi_i^*|$ from the point $\phi = 0$ exponentially decreases with each new oscillation. Since the velocity of the field at the point $\phi = 0$ remains almost unchanged until ϕ loses its energy to the created particles, the duration of each oscillation decreases exponentially as well. Therefore the whole process takes a time $\mathcal{O}(10)\phi_1^*/v$, after which the backreaction of the created particles becomes important, and the field falls to the ESP.

This process is very similar to the last stages of preheating, as studied in [1]. The main difference is that in the simplest models of preheating the field oscillates near the minimum of its classical potential. In our case the effective potential is initially absent, but a potential is generated due to the created particles. This is exactly what happens at the late stages of preheating, when the effective potential (with an account taken of the produced particles) becomes dominated by the rapidly-growing term proportional to $|\phi|$; see the discussion in section VIII B of [1].

We would like to emphasize that until the very last stages of the process, the backreaction of the created particles can be studied by the simple methods described above. At this stage the total number of created particles is still very small, but their number grows exponentially with each new oscillation. This leads to an exponentially rapid increase of the steepness of the potential energy of the field ϕ (3.15) and, correspondingly, to an exponentially rapid decrease of the amplitude of its oscillations. This extremely fast trapping of ϕ happens despite the fact that at this first stage of oscillations the total energy of ϕ, including its potential energy, remains almost constant.

Once the amplitude of oscillations becomes smaller than the width of the nonadiabaticity region, $|\phi(t)| \lesssim \Delta\phi \sim \sqrt{v/g}$, one can no longer assume that the number of particles will continue to grow via a rapidly-developing parametric resonance. The amplitude of the oscillations is given by $\frac{v^2}{2g|\phi|n_\chi}$, so the amplitude becomes $\mathcal{O}(\sqrt{v/g})$ when the total number of the produced particles grows to

$$n_\chi \sim v^{3/2} g^{-1/2}. \tag{3.20}$$

Note that the typical energy of each χ particle at $|\phi(t)| \sim \sqrt{v/g}$ is of the same order as its kinetic energy $\mathcal{O}(\sqrt{gv})$. One can easily see that the total energy density of particles χ at that stage is roughly $\sqrt{gv}n_\chi \sim \mathcal{O}(v^2)$, i.e. it is comparable to the initial kinetic energy of ϕ.

Thus, our estimates indicate that the regime of the broad parametric resonance ends when a substantial part of the initial kinetic energy of ϕ is converted to the energy of the χ particles, and the amplitude of the oscillating field ϕ becomes comparable to the width of the nonadiabaticity region,

$$|\phi| \sim \Delta\phi = \sqrt{\frac{v}{g}}\,. \tag{3.21}$$

We will use these estimates in our discussion of the cosmological consequences of moduli trapping. In order to obtain a more complete and reliable description of the last stages of this process one should use lattice simulations, taking into account the rescattering of created particles [26, 27]. An investigation of a similar situation in the theory of preheating has shown that rescattering makes the process of particle production more efficient. This speeds up the last stages of particle production and leads to a rapid decay of the field ϕ [29], which in our case corresponds to a rapid descent of ϕ toward the enhanced symmetry point.

4. Trapped moduli in an expanding universe

4.1 Rapid trapping

In this section we will study the conditions under which the trapping mechanism in quantum field theory survives the effects of coupling to gravity in an expanding universe.

First, we should point out one very beneficial effect of cosmological expansion. The field-theoretic mechanism presented above often leads to moduli being trapped in large-amplitude fluctuations (2.12) around an ESP when $\mu \neq 0$. On timescales where the expansion is noticeable, Hubble friction will naturally extract the energy from this motion, drawing the modulus inward and leading the modulus to come to rest at the ESP.

Let us now ask whether the expansion of the universe can impede moduli trapping. Consider a system of moduli coupled to gravity, with the fields arranged to roll near an ESP. For simplicity we will consider FRW solutions with flat spatial slices,

$$ds^2 = dt^2 - a(t)^2 d\vec{x}^2\,. \tag{4.1}$$

The Friedman equation determining $a(t)$ is

$$3H^2 = \frac{1}{M_p^2}\rho \tag{4.2}$$

where $H = \dot{a}/a$ and ρ is the energy density of the moduli.

The trapping effect will be robust against cosmological expansion if the timescale governing trapping is short compared to H^{-1}, i.e. if $H \ll v/\phi_*$, where ϕ_* is given by (2.12). Assuming that the potential energy of the moduli is non-negative, this implies that

$$\phi_* \ll \sqrt{6}M_p \to \frac{4\pi^3}{6^{1/2}g^{5/2}}\frac{v^{1/2}}{M_p}e^{\pi g\mu^2/v} \ll 1\,. \tag{4.3}$$

This condition suffices to ensure that trapping is very rapid.

If this condition is satisfied, trapping occurs in much less than a Hubble time, in which case the analysis of section 2 and section 3 remains valid. We will show in section 4.3 that even when (4.3) is not satisfied, trapping does still occur, although with somewhat different dynamics.

4.2 Scanning range in an expanding universe

An important effect of the gravitational coupling is that during the expansion of the universe, the energy density in produced χ particles dilutes like $1/a^3$ if they are non-relativistic and like $1/a^4$ if they are relativistic. The energy in coherent motion of ϕ, however, has the equation of state $p = \rho$ and therefore dilutes much faster, as $1/a^6$.

This effect reduces the range of motion for the moduli even before they encounter any ESPs. Hubble friction slows the progress of any rolling scalar field, and if the distance between ESPs is sufficiently large then a typical rolling modulus will come to rest without ever passing near an ESP. In order to apply our results to the vacuum selection problem, we will need to know how large a range of ϕ we can scan over in the presence of Hubble friction. This can be obtained as follows [30].

If we are in an FRW phase,

$$a(t) = a_0 t^\beta \tag{4.4}$$

then the equation of motion for ϕ (ignoring any potential terms)

$$\ddot{\phi} + 3H\dot{\phi} = 0 \tag{4.5}$$

has solutions of the form

$$\dot{\phi}(t) = v\left(\frac{t_0}{t}\right)^{3\beta} . \tag{4.6}$$

We can integrate this to determine how far the field rolls before stopping.

Let us first consider the case $\beta = 1/3$, which corresponds to the equation of state $p = \rho$. This includes the case where the coherent, classical kinetic energy of ϕ drives the expansion. The value of ϕ,

$$\phi(t) = v t_0 \log\left(\frac{t}{t_0}\right) \tag{4.7}$$

diverges at large t. Thus ϕ can travel an arbitrarily large distance in moduli space.

In the more general case $\beta > 1/3$ the field will travel a distance

$$\phi(t) - \phi(t_0) = \frac{v}{H(t_0)}\frac{\beta}{3\beta - 1} \tag{4.8}$$

before stopping.

In order to be in a phase with $\beta > 1/3$, the kinetic energy of ϕ must not be totally dominant; that is, we must have $\frac{1}{2}\dot{\phi}^2 < \rho$, where $\rho \equiv 3M_p^2 H^2$ is the total energy density appearing on the right hand side of the Friedman equation. Plugging this into (4.8) we obtain the constraint

$$\phi(t) - \phi(t_0) < \sqrt{6} M_p \frac{\beta}{3\beta - 1} . \tag{4.9}$$

Let us consider a specific example. Suppose that we start at t_0 with kinetic energy domination: $K_0/\rho_0 = 1-\epsilon$, $\epsilon \ll 1$, in some region of the universe that can be modelled as an expanding FRW cosmology. The kinetic energy drops like $K \sim \rho_0(a_0/a)^6 \sim \rho_0(t_0/t)^2$, while the other components of the energy dilute like

$$\rho(t) = \epsilon\rho_0(t_0/t)^{1+w}\,, \tag{4.10}$$

with $w < 1$. The universe will stop being kinetic-energy dominated at the time $t_c = t_0\epsilon^{-1/(1-w)}$, at which point, according to (4.7), the modulus has travelled a distance

$$\phi(t_c) - \phi(t_0) = -\frac{1}{1-w} v t_0 \log\epsilon\,. \tag{4.11}$$

After this the field keeps moving and covers an additional range

$$\phi(t_*) - \phi(t_c) = \sqrt{3} M_p \frac{2}{3(1-w)}\,. \tag{4.12}$$

To get a feel for the numbers, consider the case where $vt_0 \sim M_p$, $\epsilon \sim 10^{-2}$, and $w = 0$. Then ϕ will travel a total distance $\phi(t_*) - \phi(t_0) \sim 6M_p$ in field space, which is not particularly far. However, as we will discuss in section 6, certain moduli spaces of interest have a rich structure on sub-Planckian scales, so in these cases there is a good chance that the modulus will encounter an ESP and get trapped before Hubble friction brings the system to rest.

There is another natural possibility if we assume low-energy $\mathcal{N} = 1$ supersymmetry. If the moduli acquire their potentials from supersymmetry breaking then there is a large ratio between the Planck scale and the scale of these potentials, leading to significant scanning ranges. Specifically, consider a contribution to the energy density coming from a potential energy V at the supersymmetry-breaking scale. If the initial kinetic energy of the moduli is Planckian and the supersymmetry-breaking scale is TeV then there will be a prolonged phase in which kinetic energy dominates, since $\epsilon = V/M_p^4 \sim 10^{-64}$. This allows ϕ to scan a significantly super-Planckian range in field space.

4.3 Trapping in an expanding universe

We are now in a position to combine all the relevant effects and consider trapping during expansion of the universe. For simplicity, we will concentrate on the case of effectively one-dimensional motion, $\mu \ll \sqrt{v/g}$. Suppose that, taking into account Hubble friction, the modulus field passes in the vicinity of the ESP at some moment t_0, so that χ particles are produced, with $n_\chi(t_0) = (gv)^{3/2}/(2\pi)^3$. We will now determine the remaining evolution including both our trapping force and Hubble friction. After the particles have been produced, the field ϕ becomes attracted toward $\phi = 0$ by a force gn_χ, so taking into account the dilution of the produced particles, for $\phi > 0$ the equation of motion is

$$\ddot{\phi} + 3H\dot{\phi} = -gn_\chi(t_0)\left(\frac{a(t_0)}{a(t)}\right)^3\,. \tag{4.13}$$

For the general power law case, $a(t) \propto t^\beta$, this becomes

$$\ddot{\phi} + 3\frac{\beta}{t}\dot{\phi} = -gn_\chi(t_0)(t_0/t)^{3\beta} . \tag{4.14}$$

The general solution of this equation is

$$\phi(t) = \phi(t_0) + c(t_0^{-3\beta+1} - t^{-3\beta+1}) + \frac{gn_\chi(t_0)t_0^2}{(2-3\beta)} - \frac{gn_\chi(t_0)}{(2-3\beta)}(t_0/t)^{3\beta}t^2 \tag{4.15}$$

where c is some constant. In the important case $\beta = 2/3$, which corresponds to a universe dominated by pressureless cold matter, the general solution is

$$\phi = \phi(0) + c(t_0^{-1} - t^{-1}) - gn_\chi t_0^2 \log\frac{t}{t_0} . \tag{4.16}$$

According to these solutions, in a universe dominated by matter with non-negative pressure (i.e. $\beta \leq 2/3$) the field ϕ moves to $-\infty$ as $t \to \infty$.

Of course, as soon as the field reaches the point $\phi = 0$, this solution is no longer applicable, since the attractive force changes its sign (the potential is proportional to $|\phi|$). The result above simply means that the attractive force is always strong enough to bring the field back to the point $\phi = 0$ within finite time. Then the field moves further, with ever decreasing speed, turns back again, and returns to $\phi = 0$ once again. The amplitude of each oscillation rapidly decreases due to the combined effect of the Hubble friction and of the (weak) parametric resonance. This means that once ϕ passes near the ESP, its fate is sealed: eventually it will be trapped there.

4.4 Efficiency of trapping

It is useful to determine what fraction of all initial conditions for the moving moduli lead to trapping. There are several constraints to be satisfied. First of all, if the impact parameter μ is much larger than $\sqrt{v/g}$, the number of produced particles will be exponentially small, and the efficiency of trapping will be exponentially suppressed. Of course, eventually ϕ will fall to the enhanced symmetry point, but if this process takes an exponentially large time, the trapping effect will be of no practical significance. Thus one can roughly estimate the range of interesting impact parameters to be $\mathcal{O}(\sqrt{v/g})$.

Another constraint is related to the fact that even if initially the energy density of the universe was dominated by the moving moduli, as discussed in section 4.2, these fields can only move the distance given by (4.11), (4.12). This distance depends on the initial ratio $1-\epsilon$ of kinetic energy to total energy, leading to a scanning range CM_p in field space, where the prefactor C is logarithmically related to ϵ.

Thus, the field becomes trapped only if there is an enhanced symmetry point inside a rectangle with sides of length CM_p along the direction of motion and width $\mathcal{O}(\sqrt{v/g})$ in the direction perpendicular to the motion.

Interestingly, the total area (phase space) of the moduli trap

$$S_{\rm trap} \sim CM_p\sqrt{\frac{v}{g}} \tag{4.17}$$

increases as the coupling decreases. This implies that the efficiency of trapping grows at weak coupling. Although this may seem paradoxical, it happens because the mass of the χ particles is proportional to the coupling constant and (fixing the other parameters) it is easier to produce lighter particles. On the other hand, if g becomes too small, the trapping force $gn_\chi \sim g^{5/2}v^{3/2}$ becomes smaller than the usual forces due to the effective potential, which we assumed subdominant in our investigation.

So far we have studied the simplest model where only one scalar field becomes massless at the enhanced symmetry point. Let us suppose, however, that N fields become massless at the point $\phi = 0$. If these fields interact with ϕ with the same coupling constant g, then particles of each of these fields are produced, and the trapping force becomes N times stronger. In other words, the trapping force is proportional to the degree of symmetry at the ESP.

5. String theory effects

It is interesting to ask if there is any controlled situation where string-theoretic effects become important for moduli trapping. Here we will simply list several circumstances in which stringy and/or quantum gravity effects can come into play, as well as some constraints on these effects, leaving a full analysis of this subtle and interesting situation for the future.

5.1 Large χ mass

One way stringy and quantum gravity effects could become important in the colliding D-brane case is if the χ mass at the turnaround point is greater than string scale, $g\phi_* > m_s$. This can happen even if the velocity is so small that during the non-adiabatic period near the origin only unexcited stretched strings are created. Then, as in our above field theory analysis, we have

$$g\phi_* = \frac{4\pi^3}{g^{3/2}} v^{1/2} e^{\pi g \mu^2/v} \,. \tag{5.1}$$

In this case, the full system includes modes, namely the created χ strings, which are heavier than the string oscillator mode excitations on the individual branes. This means that the system as a whole cannot consistently be captured by pure effective field theory. However, it may still happen that the created stretched strings are relatively stable against annihilation or decay into the lighter stringy modes. Their annihilation cross section is suppressed by their large mass, as discussed in appendix B.[6] Furthermore, an individual stretched string will not directly decay if it is the lightest particle carrying a conserved charge.

This latter situation happens in the simplest version of a D-brane collision. The created stretched string cannot decay into lighter string or field theory modes because it is charged and they are not.

[6] For stringy densities of stretched strings, there could be additional corrections to the annihilation rate, but we will not consider this possibility.

5.2 Large v and the Hagedorn density of states

If we increase the field velocity $\dot{\phi} = v$, then we may obtain a situation in which excited string states are produced as ϕ passes the ESP. The number of string states produced in this process is enhanced by the Hagedorn density of states, so the Bogoliubov coefficients have the structure

$$|\beta_k|^2 = \sum_n e^{\frac{\sqrt{n}}{2\pi\sqrt{2}}} e^{-\pi(k^2+nm_s^2+g^2\mu^2)/(gv)} \tag{5.2}$$

where in the D-brane context, $g = \sqrt{g_s}$ is the Yang-Mills coupling on the D-branes. Because of the $e^{-\pi n m_s^2/(gv)}$ suppression in the second factor, this effect is only significant if $gv \gg m_s^2$.

However, in the case of colliding D-branes, and any situation dual to it, there is a fundamental bound on the field velocity from the relativistic speed limit of the branes. That is, for large velocity one must include the full Dirac-Born-Infeld lagrangian for ϕ, which takes the form

$$S = -\frac{1}{(2\pi)^3 g_s \alpha'^2} \int d^4x \sqrt{1 - g^2 \frac{\dot{\phi}^2}{m_s^4}}\,. \tag{5.3}$$

This action governs the nontrivial dynamics of ϕ for velocities approaching the string scale, and in particular, it reflects the fact that the brane velocity $g\dot{\phi}\alpha'$ must be less than the speed of light in the ambient space. Applied to our situation, (5.3) implies that the D-brane velocity cannot be large enough for the Hagedorn enhancement (5.2) to substantially increase the trapping effect.

However, in the presence of a large velocity, the effective mass of the stretched string also has important velocity-dependent contributions [5]. This may enhance the non-adiabaticity near the origin and thus enhance the particle production effect. If anything, we expect this to increase the trapping effect; it would be interesting to study this case further. As we discuss at the end of appendix A, by using unitarity combined with the stringy calculations in [23], it might be possible to determine the net result of all these effects on the stringy trapping mechanism. It would be interesting if, taking into account all relevant quantum effects, a large contribution to string production occurred in some controlled setting, as suggested in [31, 32, 33]. If such an effect occurred for motion near an ESP, it would apparently enhance the trapping effect and possibly even indicate that ϕ slows down enormously before passing through the origin, as occurred in the case studied in [5]. However, it would be important to check for control of the quantum corrections to such a system.

5.3 Light field-theoretic strings

A further possibility is to formally reduce the tension of strings by considering strings in warped throats, strings from branes partially wrapped on shrinking cycles, and the like. In these situations, the strings are essentially field-theoretic, though string theory techniques such as AdS/CFT and "geometric engineering" of field theories may provide technical help in analyzing the situation.

JHEP05(2004)030

6. The vacuum selection problem

We can now apply the ideas of the previous five sections to the cosmology of theories with moduli.

A natural application of the moduli trapping effect is to the problem of vacuum selection. One mechanism of vacuum selection is based on the dynamics of light scalars during inflation. Moduli fields experience large quantum fluctuations during inflation and can easily jump from one minimum (or valley) of their effective potential to another. It was suggested long ago that such processes may be responsible, e.g., for the choice of the vacuum state in supersymmetric theories [34] and for the smallness of the cosmological constant [35]. The probability of such processes and the resulting field distribution depends on the details of the inflationary scenario and the structure of the effective potential [36].

The mechanism that we consider in this paper is, in a certain sense, complementary to the inflationary mechanism discussed above. During inflation the average velocities of the fields are very small, but quantum fluctuations tend to take the light scalar fields away from their equilibrium positions. On the other hand, after inflation, the fields often find themselves not necessarily near the minima of their potentials or in the valleys corresponding to the flat directions, but on a hillside. As they roll down, they often acquire some speed along the valleys, see e.g. [3]. At this stage (as well as in a possible pre-inflationary epoch) the moduli trapping mechanism may operate.

This mechanism may reduce the question of how one vacuum configuration is selected dynamically out of the entire moduli space of vacua to the question of how one ESP is selected out of the set of all ESPs. This residual problem is much simpler because ESPs generically comprise a tiny subset of the moduli space.

6.1 Vacuum selection in Quantum Field Theory

In pure quantum field theory, discussed in section 2, we saw that if a scalar field ϕ is initially aimed to pass near an ESP, then ϕ gets drawn toward the ESP and is ultimately trapped there. This appears to be a basic phenomenon in time-dependent quantum field theory: moduli which begin in a coherent classical motion typically become trapped at an ESP. This leads to a dynamical preference for ESPs.

In many of the supersymmetric quantum field theories that have been studied rigorously [7], the moduli space contains singular points at which light degrees of freedom emerge. We have seen that moduli can become trapped near these points given suitable initial conditions.

6.2 Vacuum selection in supergravity and superstring cosmology

Compactifications of M/string theory which have a description as a low energy effective supersymmetric field theory can have a natural separation of scales: the string or Planck scale can be much larger than the energy scales in the effective field theory potential. Thus, the intrinsically stringy effects of section 5 are unimportant in this limit. On the other hand, the effects of coupling to gravity given in section 4 continue to provide a crucial constraint, as we will now discuss.

JHEP05(2004)030

First of all, as in the case of pure quantum field theory, there exist very instructive toy models with extended supersymmetry, for which there is no potential at all on the moduli space. For these examples, in situations where higher-derivative corrections to the effective action are suppressed, a rolling scalar field has the equation of state $p = \rho$. This corresponds to the $\beta = 1/3$ case (4.7) of section 4, for which one can scan an arbitrarily large distance in field space. Therefore, in this case, the trapping effect applies in a straightforward way to dynamically select the ESPs for regimes in which (4.3) is also satisfied.

More generally, however, one may wish to implement cosmological trapping in theories with some potential energy. In this case the requirement that the scanning range of ϕ (as constrained by Hubble friction in section 4) should be large enough to cover multiple vacua is an important constraint. The absolute minimum requirement is that the scanning range is sufficient for the moduli to reach one ESP before stopping from Hubble friction; but to address the vacuum selection problem one should ideally scan a number of ESPs.

One context in which this can happen is in a phase in which the kinetic energy of the rolling scalar fields dominates the energy density of the universe so that the $\beta = 1/3$ result (4.7) applies. This may occur in a pre-inflationary phase in some patches of spacetime, though it is subject to the stringent limitation in duration given in (4.11). Given such a phase, the field will roll around until it gets trapped at an ESP.

During the ordinary radiation-dominated ($\beta = 1/2$) and matter-dominated ($\beta = 2/3$) eras, the more stringent constraint (4.9) applies. As we indicated in section 4, this scanning range is not large in Planck units, so we can usefully apply moduli trapping to the problem of vacuum selection in these eras only if the vacuum has appropriately rich structure on sub-Planckian scales. In other words, the average distance in moduli space between ESPs should be sub-Planckian.

Gravitationally-coupled scalars ϕ generically have a potential energy $V(\phi/M_p)$ which has local minima separated by Planck-scale distances. In this cases, the limited scanning range during the $\beta \neq 1/3$ cosmological eras prevents our mechanism from addressing the vacuum selection problem. However, it is generic for compactification moduli to have special ESPs where the gravitationally-coupled system is enhanced to a system with light field theory degrees of freedom. Given a rich enough effective field theory in this ESP region, there will generically be interesting vacuum structure on sub-Planckian distances. In this sort of region moduli trapping will pick out the ESP vacua of the system.

6.3 Properties of the resulting vacua

Let us now consider the qualitative features of the vacua selected by moduli trapping, assuming that the constraint imposed by Hubble friction has been evaded in one of the ways described above.

First of all, it is important to recognize that what we have called ESPs may well be subspaces of various dimensions, not points. For example, in toroidal compactification of the heterotic string, there is one enhanced symmetry locus for each circle in the torus — new states appear when the circle is at the self-dual radius. Each of these loci is codimension one in the moduli space, but of course their intersections, where multiple radii are self-dual, have higher codimension.

When moduli trapping acts in such a system of intersecting enhanced symmetry loci, we expect that the moduli will first become trapped on the locus of lowest codimension, but retain some velocity parallel to this locus. Further trapping events can then localize the modulus to subspaces of progressively higher codimension. The final result is that the moduli come to rest on a locus of maximally enhanced symmetry.

The simplest examples of this phenomenon are toroidal compactification, in which all circles end up at the self-dual radius, and the system of N D-branes discussed in section 2.3, in which the gauge symmetry is enhanced to $\mathrm{U}(N)$.[7]

Quite generally, we expect that within the accessible range in field space, taking into account Hubble friction and the form of the potential, moduli trapping will select the ESPs with the largest number of light states, which often corresponds to the highest degree of symmetry.[8]

In some very early epoch the rolling moduli can have large velocities, so trapping can occur even at points where the "light" states χ have a relatively large mass, and the enhanced symmetry is strongly broken. However, Hubble friction inevitably slows the motion of the moduli. Thus, trapping at late times is possible only at ESPs with weakly-broken symmetries and very light particles. One could speculate about a possible relation of this fact to the mass hierarchy problem.

Note that even though we emphasized the natural role of enhanced symmetry in moduli trapping, in fact the only strict requirement was the appearance of new light particles at the trapping points. In some of the many vacua of string theory, particles may be light not because of symmetry but because of some miraculous cancellations. Invoking such unexplained cancellations to produce a small mass is highly undesirable. However, moduli trapping may ameliorate this problem, as those rare points in moduli space where the cancellation does happen are actually dynamical attractors.

Thus, the attractive power of symmetry and of light particles may have implications for questions involving the distribution of vacua in string theory [37, 38, 39, 40]. Given the strong preference we have seen for highly-enhanced symmetry, the distribution of all string vacua obtained by a naive counting, weighted only by multiplicity, may be quite different from the distribution of vacua produced by the dynamical populating process discussed in our paper. It is therefore very tempting to speculate that some of the surprising properties of our world, which might seem to be due to pure chance or miraculous cancellations, in fact may result from dynamical evolution and natural selection.

7. The moduli problem

One aspect of the moduli problem is that reheating and nucleosynthesis can be corrupted

[7] A toy model for this situation, in the case of three D-branes, has the potential $\frac{g^2}{2}[\chi_1^2|\phi_2-\phi_3|^2+\chi_2^2|\phi_1-\phi_3|^2+\chi_3^2|\phi_1-\phi_2|^2]$, where ϕ_i and χ_i are six different fields. Suppose that ϕ_2 moves through the point $\phi_2-\phi_3=0$. This creates χ_1 particles and traps the system at $\phi_2=\phi_3$, where χ_1 is massless. Subsequent motion of ϕ_1 can trap it at the point $\phi_1=\phi_2=\phi_3$, making the remaining fields χ_2 and χ_3 massless.

[8] Moreover, as we discuss in appendix B, the trapping effect is far more effective at ESPs for which the χ particles do not decay rapidly. We therefore expect moduli trapping to select ESPs which have relatively stable light states.

by energy locked in oscillations of the moduli. The source of the problem is that the true minima of the low-temperature effective potential applicable after inflation do not coincide with the minima of the Hubble-temperature effective potential which is valid during inflation. It follows that moduli which sit in minima of the latter during inflation will find themselves displaced from their true, low-temperature minima once inflation is complete. The energy stored in this displacement, and in the resulting oscillations about the true minimum, poses problems for nucleosynthesis.

One way to address this problem is to permit initial displacements of the moduli, as described above, but somehow arrange that the oscillating moduli decay very rapidly to Standard Model particles. Alternatively, one could fix the moduli at a scale high enough that the Hubble temperature during inflation does not destabilize them. This may work in string models with stabilized moduli such as [38, 41, 39, 42].

Another approach to this problem [11] is to posit that the moduli sit at an enhanced symmetry point minimum of the finite-temperature effective potential during inflation. Then, when inflation ends, the moduli are still guaranteed to be at an extremum of the effective potential. If this extremum is a minimum then the moduli have no problematic oscillations after inflation. Our trapping mechanism allies nicely with this idea by providing a preinflationary dynamical mechanism which explains the initial condition assumed in this scenario. That is, in parts of the universe where ϕ kinetic energy dominates well before inflation, the trapping effect can explain why the moduli find themselves in ESP minima at the onset of inflation.

8. Trapped inflation and acceleration of the universe

The main motivation of our investigation was to study the behavior of moduli in quantum field theory and string theory. However, the results we have obtained have more general applicability. To give an example, in this section we will study the cosmological implications of the trapping of a scalar field ϕ with a relatively steep potential.

Consider the theory of a real scalar field ϕ with the effective potential $m^2\phi^2/2$. In the regime $\phi < M_p$ the curvature of the effective potential is greater than H^2, with H the Hubble parameter, so ϕ falls rapidly to its minimum, and inflation does not normally occur.

We will assume that ϕ gives some bosons χ a mass $g|\phi - \phi_1|$. Let us assume that ϕ falls from its initial value $\phi_0 = \phi_1(1+\alpha) < M_p$ with vanishing initial speed. If we take $\alpha \ll 1$ and neglect for the moment the expansion of the universe, then ϕ arrives at ϕ_1 with the velocity $v = \sqrt{2\alpha}\, m\phi_1$.

As ϕ passes ϕ_1, it creates χ particles with number density $n_\chi = (gv)^{3/2}/8\pi^3$. After a very short time these particles become nonrelativistic, and further motion of ϕ away from ϕ_1 requires an energy $g|\phi - \phi_1| n_\chi$. In other words, the effective potential becomes

$$V(\phi) \approx \frac{1}{2}m^2\phi^2 + gn_\chi|\phi - \phi_1| = \frac{1}{2}m^2\phi^2 + g^{5/2}\frac{v^{3/2}}{8\pi^3}|\phi - \phi_1|\,. \tag{8.1}$$

For $g^{5/2}\frac{v^{3/2}}{8\pi^3 m^2} > \phi_1$, the minimum of the effective potential is not at $\phi = 0$, but at the point ϕ_1, where the particle production takes place. The condition $g^{5/2}\frac{v^{3/2}}{8\pi^3 m^2} > \phi_1$ implies that

$$m < 2^{-9/2}\pi^{-6} g^5 \alpha^{3/2} \phi_1 \,. \tag{8.2}$$

Thus, if the mass of ϕ is sufficiently small, the field will be trapped near the point ϕ_1.

To give a particular example, take $\phi_1 \sim M_p/2$, $\alpha \sim 1/4$. Then ϕ is trapped near ϕ_1 if

$$m < 10^{-6} g^5 M_p \,. \tag{8.3}$$

For a very light field, such as a modulus with $m \sim 10^2$ GeV $\sim 10^{-16} M_p$, this condition is readily satisfied unless g is very small.

Once the field is trapped, it starts oscillating around ϕ_1 with ever-decreasing amplitude, creating new χ particles in the regime of parametric resonance. Eventually ϕ transfers a large fraction of its energy to χ particles. One can easily check that in this model the fall of ϕ to the point ϕ_1 and the subsequent process of creation of χ particles occurs within a time smaller than H^{-1}, so one can neglect expansion of the universe at this stage. This process is therefore governed by the theory described in section 3. In particular, we may use the estimate (3.20) of the total number of χ particles produced in the process. At the end of the particle production, the correction to the effective potential becomes much larger than at the beginning of the process:

$$\Delta V = g|\phi - \phi_1| n_\chi \sim v^{3/2} g^{1/2} |\phi - \phi_1| \,, \tag{8.4}$$

Subsequent expansion of the universe dilutes the density of χ particles as a^{-3}, which eventually makes the correction to the effective potential small, so that ϕ starts moving down again. The field ϕ remains trapped at $\phi = \phi_1$ until the scale factor of the universe grows by a factor

$$a \sim \alpha^{1/4} \left(\frac{g\phi_1}{m}\right)^{1/6} \tag{8.5}$$

since the beginning of the trapping process.

In the beginning of the first e-folding, the kinetic energy of the χ particles and of the oscillations of ϕ is comparable to the potential energy of ϕ. However, the kinetic energy rapidly decreases, and during the remaining time the energy is dominated by the potential energy $V(\phi_1)$. This means that the trapping of ϕ may lead to a stage of inflation or acceleration of the universe, even if the original potential $V(\phi)$ is too steep to support inflation.

Let us consider various possibilities for the scales in the potential, to get some simple numerical estimates for the duration of inflation. For example, if we take $\alpha, g = \mathcal{O}(1)$, $\phi_1 \sim M_p$ and $m \sim 10^2$ GeV, then the scale factor during a single trapping event will grow by a factor of e^6. If one considers a model with $m \sim 10^{-30} M_p$, which can arise in a radiatively stable manner (as in the "new old inflation" model [17]), the scale factor during a single trapping event can grow by a factor of e^{11}. Finally, if the moduli mass is of the same order as a typical mass taken in theories of quintessence, $m \sim 10^{-60} M_p$, we can have

JHEP05(2004)030

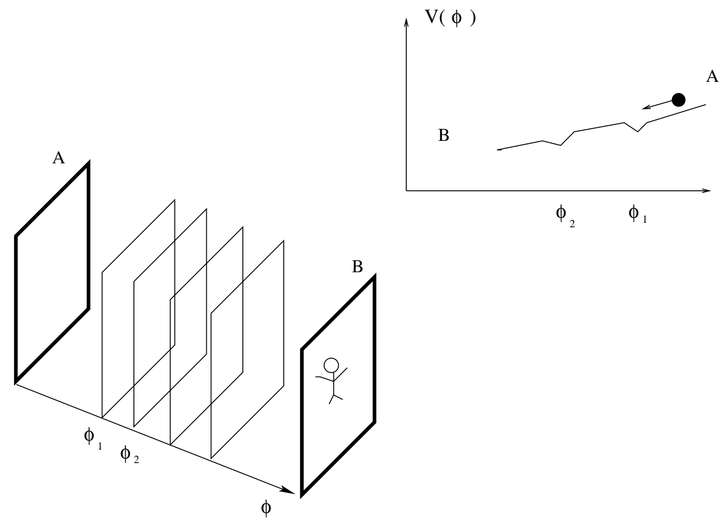

Figure 4: The D-brane picture of a series of trapping events.

an accelerated expansion of the universe by a factor e^{23}, in a sub-Planckian regime of field space, just from trapping. (In this last case, as in ordinary quintessence models, tuning is required.)

Thus, the stage of inflation in this simple model is shorter than the usual 60 e-folds, but it may nevertheless be very useful for initiating a first stage of inflation in theories where this would otherwise be impossible, or for diluting unwanted relics at the later stages of the evolution of the universe. Moreover, this scenario can easily describe the present stage of acceleration of the universe.

One can also make the effect more substantial by constructing a more complicated scenario, consisting of a chain of N particle production events at locations $\phi = \phi_i$, where some fields χ_i become light. The field ϕ may be trapped and enter the stage of parametric resonance near each of these points. Correspondingly, the universe enters the stage of inflation many times. One could arrange for 60 e-folds of inflation by taking, for example, $m \sim 10^2$ GeV, $N \sim 10$.

A D-brane example provides a useful geometrical model of this process. Suppose we have an observable brane B and another brane A approaching it. Suppose also that there are a number of other branes in between A and B. Each time the moving brane passes through one of the standing intermediate branes, stretched strings are created and slow the motion of A. The cumulative effect of a number of standing branes is perceived on the observable brane as a slowing-down of the motion of A due to the interactions.

One should note that inflation in our scenario is rather unusual: the inflaton ϕ rolls a short distance, then oscillates for a long time, but with period much smaller than H^{-1}, then rolls again, etc. This may lead to peculiar features in the spectrum of density perturbations. One can avoid these features if the points ϕ_i are very close to each other, and each of them does not stop the rolling of ϕ but only slows it down. In this case, particle production will not lead to parametric resonance, so it is not very important to us whether the fields χ_i are bosons or fermions, as long as their masses vanish at ϕ_i.

This scenario is similar to the string-inspired thermal inflation considered in [12] (see also [43]), but our proposal does not require thermal equilibrium. The main effect which supports inflation in our scenario is based on particle production and has a nonperturbative origin. (A closely-related mechanism uses the corrections to the kinetic terms in the strong coupling regime, where the particle production is suppressed [5].) We hope to return to a discussion of this possibility in a separate publication.

9. Conclusion

We have argued that the dynamics of rolling moduli is considerably modified due to quantum production of light fields. In flat space quantum field theory, moduli typically become trapped in orbits around loci which have extra light degrees of freedom. In the presence of gravity, Hubble friction limits the field range the system samples, but any trapping events which do occur are enhanced by Hubble friction, which rapidly brings the modulus to rest at an ESP. Moduli trapping may aid in solving the cosmological moduli problem by driving moduli to sit at points of enhanced symmetry. Furthermore, the trapping of a scalar field which has a potential can lead to a short period of accelerated expansion in situations with steeper potentials than would otherwise allow this. Finally, the trapping effect has important consequences for the problem of vacuum selection, as it can reduce the problem to that of selecting one point within the class of ESPs. An intriguing feature of this process is that the trapping is more efficient near points with a large number of unbroken symmetries.

Acknowledgments

We would like to thank M. Dine, G. Felder, S. Kachru, R. Kallosh, R. Myers, S.-J. Rey, S. Shenker, D. Tong, and A. Tseytlin for useful discussions. L.K. is supported by NSERC and CIAR. The remaining authors are supported in part by the DOE under contract DE-AC03-76SF00515 and by the NSF grant PHY-0244728.

A. Particle production due to motion on moduli space

In this section we will calculate the quantum production of χ particles, ignoring the effect of backreaction on the motion of ϕ.

A mode of χ with spatial momentum k obeys the wave equation

$$\left(\partial_t^2 + k^2 + g^2(\mu^2 + v^2 t^2)\right) u_k = 0\,. \tag{A.1}$$

There are two solutions to this equation, $u_k^{\rm in}$ and $u_k^{\rm out}$, associated to vacuum states with no particles in the far past and no particles in the far future, respectively. These two sets of modes are related by a Bogoliubov transformation

$$u_k^{\rm in} = \alpha_k u_k^{\rm out} + \beta_k u_k^{\rm out *}\,. \tag{A.2}$$

If we start in the state with no particles in the far past, then one can calculate the number density of particles in the far future to be

$$n_k = |\beta_k|^2 \tag{A.3}$$

in the k^{th} mode. This may be evaluated by solving equation (A.1) in terms of hypergeometric functions (see e.g. section 3.5 of [13]), but we will present here a more physical argument.

One can view (A.1) as a one dimensional Schrödinger equation for particle scattering/penetration through an inverted parabolic potential. If we send in a wave $\psi_k^{\rm in}$ from the far right of the potential, part of it will penetrate to the far left, with an asymptotic amplitude $T_k\psi_k^{\rm out}$, and part of it will be reflected back to the right, with an asymptotic amplitude $R_k\psi_k^{in*}$, where T_k and R_k are the transmission and reflection amplitudes.[9]

The Bogoliubov coefficient in (A.2) is determined in terms of these transmission and reflection amplitudes via

$$\beta_k = \frac{R_k^*}{T_k^*}\,. \tag{A.4}$$

Now we use a trick from quantum mechanics to relate R and T using the WKB method. If we are moving along the real time coordinate, the WKB form of the solution $u_k^{\rm in}(t)$ will be violated at small t, due to non-adiabaticity. However, if we take t to be complex then we can move from $t = -\infty$ to $t = +\infty$ along a complex contour in such a way that the WKB approximation

$$u_k^{\rm in}(t) \sim \frac{1}{\sqrt{2\sqrt{k^2+g^2(\mu^2+v^2t^2)}}}e^{-i\int^t\sqrt{k^2+g^2(\mu^2+v^2t'^2)}dt'} \tag{A.5}$$

is valid. Here the integral $\int^t dt'$ becomes a contour integral along a semicircle of large radius in the lower complex t plane. For large $|t|$, we can estimate the phase integral in (A.5) by expanding

$$\sqrt{k^2+g^2(\mu^2+v^2t^2)} \sim gvt + \frac{k^2+g^2\mu^2}{2gvt}\,. \tag{A.6}$$

As we go around half of the circle, this term generates a factor

$$(e^{-i\pi})^{-i(k^2+g^2\mu^2)/2gv-1/2} = ie^{-\pi(k^2+g^2\mu^2)/2gv}\,. \tag{A.7}$$

This is exactly the ratio between R^* and T^*, so we find

$$n_k = |\beta_k|^2 = e^{-\pi(k^2+g^2\mu^2)/gv}\,. \tag{A.8}$$

It is important to note that this result applies much more generally than for

$$\phi = i\mu + vt\,. \tag{A.9}$$

In many cases the nonadiabaticity is only appreciable near the origin $\phi = 0$, so that the near-origin trajectory can be approximated by (A.9) with some appropriate near-origin velocity v, even if the evolution away from the origin is very different from (2.2).

[9]The modes in the two problems are related by $u_k^{\rm in}(t\to-\infty) = T_k^*\psi_k^{\rm out*}$, etc.

Moreover, in analogous circumstances (T-dual in the brane context) with a nontrivial electric field, we obtain a similar expression due to Schwinger pair production; a related point was made in [23]. In addition, formula (A.8) applies not only to scalar fields, but also to fields of arbitrary spin. From this universal behavior, it is tempting to speculate that (A.8) could provide an effective model for string theory effects, but we will not pursue this direction here.

The result (A.8) is nonperturbative in g (with g, not g^2, appearing in the denominator of the exponent); it is interesting to ask whether there is a simple interpretation of this nonanalytic, nonperturbative effect. Similarly, it is interesting to note that as discussed in section 3, the potential for ϕ induced by particle production is linear, so that if extended to the origin it would have a nonanalytic cusp there.

Our results correspond to the low-velocity limit of the D-brane calculation by Bachas [23]. There, he obtains an imaginary part to the action for moving D3-branes of the form

$$\mathrm{Im}\, S \propto \sum_{n=1}^{\infty} \frac{(-1)^{(n+1)}}{n} \left(\frac{gv}{\pi n}\right)^{3/2} \exp\left(-\frac{n\pi g\mu^2}{v}\right) \tag{A.10}$$

where we have translated his results into our variables. The first term in this expansion is proportional to the overlap $\int d^3\vec{k}|\beta_k|^2$ giving the number density (3.14) of produced particles; this agrees with what we expect from unitarity. More generally, backing away from this low-velocity limit, the calculation in [23] combined with unitarity may provide a simple generalization of our results to the string case, as we briefly discussed in section 5.2.

B. Annihilation of the χ particles

In this section we study the effects of collisions and direct decays of the created χ particles, and demonstrate that for suitably chosen parameters the trapping effect receives only small corrections. More specifically, we place limits on the reduction of the χ number density through processes like $\chi\chi \to \phi\bar{\phi}$ and $\chi \to \eta\bar{\eta}$, where η is some light field.

Direct decays, if present, could easily ruin the trapping mechanism: if the χ particles decay too rapidly into light fields then the energy stored in created χ particles will not suffice to stop ϕ. In this case the modulus will roll past the ESP, feel a transient tug toward the ESP while the χ particles remain, and then gradually break free and glide off to infinity at a reduced speed.

We will therefore consider only models in which couplings of the form $\chi\bar{\psi}\psi$, with $\bar{\psi}, \psi$ very light, are negligible. As an example, one can easily exclude such decays in a supersymmetric model with a superpotential of the form $\mathcal{W} \sim g\Phi\mathrm{X}^2$. Here X is a chiral superfield with scalar component χ and fermion ψ_χ, and Φ is a chiral superfield with scalar component ϕ and fermion ψ_ϕ. This generates Yukawa couplings of the form $\chi\psi_\chi\psi_\phi$ and $\phi\psi_\chi\psi_\chi$, which do not allow decays from a component of X to purely Φ particles. Thus, if all components of X are heavy, the X-particle energy density we produce cannot decrease by direct decays. In some of the simplest brane setups, exactly this situation is realized: a string which is heavy because it stretches between two branes separated in a purely closed string bulk space cannot decay perturbatively into two light, unstretched strings.

On the other hand, a priori we cannot ignore the coupling $\frac{g^2}{2}\chi^2\phi^2$ as it is this which gives rise to the desired trapping effect. This means that we must tolerate a certain rate of annihilation (as opposed to direct decay). We will now review the cross section for this process and determine its effect on the number density n_χ appearing in (2.10).

The Lorentz-invariant cross section for the annihilation process $\chi\chi \to \phi\bar{\phi}$ is, written in terms of center of mass variables,

$$\sigma = \frac{g^4 k'}{4\pi k E^2}\,, \tag{B.1}$$

where k and k' are the momenta of the ingoing and outgoing particles and E is the energy of the ingoing χ particles. The reverse process $\phi\bar{\phi} \to \chi\chi$ tends to enhance the trapping effect. As we are in search of a lower bound on the number of χ particles, we will simply omit this reverse process.

We now determine the annihilation rate to find the rate at which χ particles are lost. If we assume that all the χ's are produced at $t = 0$, we find

$$\frac{\dot{n}(\vec{k}_1,t)}{n(\vec{k}_1,t)} = -\int d\vec{k}_2 n(\vec{k}_2,t)\frac{\sqrt{(k_1k_2)^2 - m_\chi^4}}{E_1E_2}\sigma(\vec{k}_1,\vec{k}_2)\,. \tag{B.2}$$

Here $u = \sqrt{(k_1k_2)^2 - m_\chi^2}/E_1E_2$ is the Lorentz-invariant relative velocity of the initial χ's and $\sigma(\vec{k}_1,\vec{k}_2)$ is the cross section, to be calculated using (B.1) in the center of mass frame.

We can simplify (B.2) to get an upper bound on how fast χ decays. Ignoring the momentum dependence on the right hand side of (B.2), which amounts to taking the non-relativistic limit, and ignoring the mass of ϕ produced by the χ particles, we have

$$\frac{\dot{n}(\vec{k}_1,t)}{n(\vec{k}_1,t)} \geq -\frac{g^4}{2\pi m_\chi^2}\int d\vec{k}_2 n(\vec{k}_2,t)\,. \tag{B.3}$$

We can bound the integral in the second term on the right hand side by n_χ, the total number of χ's produced, as given in (3.14). To approximate the time-dependence of the mass m_χ, we take $m_\chi^2 = \mu^2 + v^2t^2$, which is what the uncorrected motion for ϕ would give. So we have finally

$$\frac{\dot{n}(\vec{k}_1,t)}{n(\vec{k}_1,t)} \geq -\frac{g^4 n_\chi}{2\pi(\mu^2 + v^2t^2)}\,, \tag{B.4}$$

which yields

$$\frac{n(\vec{k},t)}{n(\vec{k},0)} \geq \exp\left(-\frac{g^4 n_\chi}{2\pi\mu v}\arctan\frac{vt}{\mu}\right). \tag{B.5}$$

This is clearly bounded from below by

$$\exp\left(-\frac{g^4 n_\chi}{2\mu v}\right). \tag{B.6}$$

so that the number density is reduced over time by at worst the factor (B.6).

The total energy density in the χ particles at a given time is therefore

$$
\begin{aligned}
E &= \int d\vec{k} n(\vec{k}, t) \sqrt{k^2 + g^2(\mu^2 + v^2 t^2)} \\
&\geq n_\chi \sqrt{g^2(\mu^2 + v^2 t^2)} \exp\left(-\frac{g^4 n_\chi}{2\pi\mu v} \arctan\frac{vt}{\mu}\right) .
\end{aligned} \tag{B.7}
$$

From this we see that the mass amplification effect of the χ particles inevitably prevails and stops ϕ from rolling arbitrarily far past the enhanced symmetry point.

This reduction of the number density softens, but does not ruin, the trapping effect. Using the energy density (B.7) in the simple estimate leading to (2.12), we find a new estimate for ϕ_*:

$$
\phi_* = \frac{4\pi^3}{g^{5/2}} v^{1/2} e^{\pi g \mu^2/v} e^{g^4 n_\chi / 4\mu v} . \tag{B.8}
$$

Thus, although collisions never lead to an escape, they do lead to a somewhat increased stopping length ϕ_*.

For suitably chosen parameters we can arrange that the effect of collisions is unimportant and the estimates (2.12), (B.8) approximately agree. For example, the final exponential factor, which encodes the consequences of annihilations, will be less important than the factor $e^{\pi g\mu^2/v}$ as long as $g^3 v \ll \mu^2$.

We conclude that direct decays can be forbidden using symmetry, whereas collisions increase the stopping length (B.8) but do not ruin the trapping effect.

C. Bending angle and energy loss during the first pass

In this section we will demonstrate that one can obtain analytical results for the early stage of the motion of the moduli through a systematic expansion and iteration procedure. Specifically, we start with the uncorrected trajectory for the moduli, work out the production of χ particles to the second order in $\dot{\omega}/\omega^2$, feed this back into the equation of motion for the modulus, and then obtain the corrected trajectory for the modulus to leading nontrivial order of the iteration and $\dot{\omega}/\omega^2$.

The solution shows deflection and energy loss, as we would expect. However, without going to higher orders in the iteration, and without being able to detect particle production which is nonperturbative in $\dot{\omega}/\omega^2$ (as in (2.9)), we do not expect our solution to capture the full dynamics. In particular, the leading-order calculations presented here do not capture the late-time behavior of the system, where any small particle production, such as that in (2.9), can alter the trajectory from an escaping one to a trapped one.

We choose to work with the simple model (2.1) in which the complex modulus $\phi = \phi_1 + i\phi_2$ has an ESP at the origin, where a real scalar field χ becomes light. Without including backreaction, the trajectory (2.2) leads to a time-varying frequency for the χ excitations,

$$
\omega_k(t) = \sqrt{k^2 + g^2(\mu^2 + v^2 t^2)} . \tag{C.1}
$$

We start by defining the generalized Bogoliubov coefficients $\alpha_{\vec{k}}(t)$ and $\beta_{\vec{k}}(t)$:[10]

$$
\begin{aligned}
u_{\vec{k}}(t) &= \frac{1}{\sqrt{2\omega_{\vec{k}}(t)}}\left(\alpha_{\vec{k}}(t)\exp\left(-i\int^t \omega_{\vec{k}}(t')dt'\right) + \beta_{\vec{k}}(t)\exp\left(+i\int^t \omega_{\vec{k}}(t')dt'\right)\right) \\
\dot{u}_{\vec{k}}(t) &= -i\sqrt{\frac{\omega_{\vec{k}}(t)}{2}}\left(\alpha_{\vec{k}}(t)\exp\left(-i\int^t \omega_{\vec{k}}(t')dt'\right) - \beta_{\vec{k}}(t)\exp\left(+i\int^t \omega_{\vec{k}}(t')dt'\right)\right). \quad \text{(C.2)}
\end{aligned}
$$

This is possible as long as $\omega_k(t)$ is never zero. The consistency of this definition combined with the Klein-Gordon equation for $u_{\vec{k}}(t)$ leads to two coupled first-order ODE's for $\alpha_{\vec{k}}(t)$ and $\beta_{\vec{k}}(t)$

$$
\begin{aligned}
\dot{\alpha}_{\vec{k}}(t) &= \frac{\dot{\omega}_{\vec{k}}(t)}{2\omega_{\vec{k}}(t)}\exp\left(+2i\int^t \omega_{\vec{k}}(t')dt'\right)\beta_{\vec{k}}(t) \\
\dot{\beta}_{\vec{k}}(t) &= \frac{\dot{\omega}_{\vec{k}}(t)}{2\omega_{\vec{k}}(t)}\exp\left(-2i\int^t \omega_{\vec{k}}(t')dt'\right)\alpha_{\vec{k}}(t)\,. \qquad \text{(C.3)}
\end{aligned}
$$

The normalization for $u_{\vec{k}}(t)$ implies

$$\alpha^2_{\vec{k}}(t) - \beta^2_{\vec{k}}(t) = 1\,. \qquad \text{(C.4)}$$

Finally, the initial condition for $\chi(t)$ as $t \to -\infty$ corresponds to

$$\alpha_{\vec{k}} = 1\,, \qquad \beta_{\vec{k}} = 0\,. \qquad \text{(C.5)}$$

When the frequency $\omega_k(t)$ changes adiabatically, $|\dot{\omega}/\omega^2| \ll 1$, the exponentials approximate the positive and negative frequency modes of the system and $\beta_k(t)$ describes particle production.

In terms of $\alpha_{\vec{k}}(t)$ and $\beta_{\vec{k}}(t)$

$$\langle\chi^2(t)\rangle = \frac{1}{4\pi^2}\int_0^\infty \frac{dkk^2}{\omega_k(t)}\left(1 + 2|\beta_k(t)|^2 + 2\,\mathrm{Re}\left(\alpha_k(t)\beta^*_k(t)\exp\left(-2i\int^t \omega_k(t')dt'\right)\right)\right). \qquad \text{(C.6)}$$

The first term in the above equation is the ordinary Coleman-Weinberg potential (after cancelling against some polynomial counterterms), and we ignore it for the reasons mentioned in section 3.2. The second and third terms encode the particle production effects; we will calculate these by an adiabatic approximation.

To the leading nontrivial order, $\alpha_k(t)$ and $\beta_k(t)$ satisfy

$$
\begin{aligned}
\dot{\alpha}_k(t) &= 0 \\
\dot{\beta}_k(t) &= \frac{\dot{\omega}_k(t)}{2\omega_k(t)}\exp\left(-2i\int^t \omega_k(t')dt'\right) \qquad \text{(C.7)}
\end{aligned}
$$

where we have used the zeroth-order Bogoliubov coefficients, $\alpha^0_{\vec{k}}(t) = 1$, $\beta^0_{\vec{k}}(t) = 0$. These are subject to the initial conditions (C.5), so that $\alpha_k(t) = 1$ and

$$\beta_k(t) = \left(\frac{i\dot{\omega}_k}{4\omega_k^2} - \frac{\dot{\omega}_k^2}{4\omega_k^4} + \frac{\ddot{\omega}_k}{8\omega_k^3} + \cdots\right)\exp\left(-2i\int^t \omega_k dt'\right). \qquad \text{(C.8)}$$

[10] Similar calculations appeared in [1].

This yields

$$\begin{aligned}\langle\chi^2(t)\rangle-\delta_M &= \frac{1}{16\pi^2}\int_0^\infty dkk^2\frac{1}{\omega_k(t)}\left(\frac{\ddot\omega_k(t)}{\omega_k(t)^3}-\frac{3\dot\omega_k(t)^2}{2\omega_k(t)^4}+\cdots\right)\\ &= \frac{v^2\mu^2}{48\pi^2(\mu^2+v^2t^2)^2}\,.\end{aligned} \tag{C.9}$$

To the first order of iteration, the corrected equation of motion for ϕ is

$$\ddot\phi+\frac{g^2v^2\mu^2}{48\pi^2(\mu^2+v^2t^2)^2}\phi=0 \tag{C.10}$$

with initial conditions

$$\phi(t_0)=vt_0+i\mu,\qquad \dot\phi(t_0)=v\,. \tag{C.11}$$

We will eventually take the initial time t_0 to be $-\infty$. Moreover, we have in mind $v_0<0$, $\mu>0$, so that ϕ comes from $\phi_1=+\infty$ with a displacement in the positive direction of ϕ_2 and shoots towards the left.

The solution for the above system is

$$\phi(t)=\mu e^{\frac{i\pi\sqrt{1+c}}{2}}\left(\left(1+\frac{1}{\sqrt{1+c}}\right)\varphi_1\left(\frac{\mu}{vt}\right)+\left(1-\frac{1}{\sqrt{1+c}}\right)\varphi_2\left(\frac{\mu}{vt}\right)\right), \tag{C.12}$$

where

$$\begin{aligned} c &= \frac{g^2}{48\pi^2}\\ \varphi_1(z) &= \frac{1}{2z}(z-i)^{\frac{1+\sqrt{1+c}}{2}}(z+i)^{\frac{1-\sqrt{1+c}}{2}}\,,\\ \varphi_2(z) &= \frac{(-1)^{-\sqrt{1+c}}}{2z}(z-i)^{\frac{1-\sqrt{1+c}}{2}}(z+i)^{\frac{1+\sqrt{1+c}}{2}}\,,\end{aligned} \tag{C.13}$$

and the branch cut goes from $z=-i$ to $z=i$.

As time goes from $-\infty$ to $+\infty$, z goes from 0^+ to ∞ and then to 0^-. We can enclose the contour by a semicircle either in the upper or lower half-plane, giving the same result:

$$\begin{aligned}\varphi_1 &\to e^{i\pi(1+\sqrt{1+c})}\varphi_1\\ \varphi_2 &\to e^{i\pi(1-\sqrt{1+c})}\varphi_2\,.\end{aligned} \tag{C.14}$$

We find that at late time

$$\begin{aligned}\phi(t\to+\infty)\to{}& -vt\Big(\cos(\pi\sqrt{1+c})+\frac{i}{\sqrt{1+c}}\sin(\pi\sqrt{1+c})\Big)-\\ &-i\mu\Big(\cos(\pi\sqrt{1+c})+i\sqrt{1+c}\sin(\pi\sqrt{1+c})\Big)+\mathcal{O}\left(\frac{\mu}{vt}\right).\end{aligned} \tag{C.15}$$

The leading large time behavior is captured by the first term, which describes a constant linear motion on the moduli space. It shows a deflection in the direction of the ESP (counterclockwise) determined by the coefficient of the leading term $(\cos(\pi\sqrt{1+c})+\frac{i}{\sqrt{1+c}}\sin(\pi\sqrt{1+c}))$. As we expect, if $g=0$ the modulus goes through a straight line from right to left. The bending angle increases monotonically, with no upper bound, as g increases. Thus, if g is very big, ϕ can orbit around the origin many times, with a growing radius, until eventually it shoots out to infinity in a fixed direction.

The modulus loses a finite fraction of its energy as it passes by the ESP. This can be seen from the fact that the coefficient of $-vt$ has norm less than one in (C.15). It so happens that at this order of iteration the fraction of energy lost depends only on the coupling g:

$$-\frac{\Delta E}{E_0} = \frac{c}{1+c}\sin^2(\pi\sqrt{1+c})\,. \tag{C.16}$$

The above picture is valid only in certain parameter regimes. Specifically, if taking only the leading nontrivial contribution to $\langle\chi^2(t)\rangle$ is to make any sense, then the non-adiabaticity parameter must be small:

$$\left|\frac{\dot{\omega}_k}{\omega_k^2}\right| \sim \frac{v}{g\mu^2} \ll 1\,. \tag{C.17}$$

This condition also implies that the effects of the produced χ particles on the motion of ϕ shows up only at exponentially late times, so that it is sensible to have an intermediate stage in which the ϕ field has a linear motion. Of course, we know this is not the full picture. The linear growth of the mass of the χ particles would eventually stop ϕ and bring it back.

D. Classical trapping versus quantum trapping

In this section we will compare our quantum trapping mechanism with the purely classical trapping proposed in [44].

Consider for simplicity a theory of two real scalar fields, ϕ and χ, with the interaction $\frac{g^2}{2}\phi^2\chi^2$. In our discussion in the main text we assumed the initial conditions $\langle\chi\rangle = 0, \langle\phi\rangle \neq 0, \dot{\chi} = 0, \dot{\phi} = v$. Potentially interesting classical dynamics arises in the more general case in which the initial velocity of χ is nonzero [44].

Let us therefore consider the classical behavior of these fields, ignoring particle production entirely. If we define $v \equiv \sqrt{\dot{\chi}^2 + \dot{\phi}^2}$, then energy conservation implies that the trajectory of ϕ and χ is bounded by the surface $g^2\phi^2\chi^2 = v^2$. The fields will evidently start bouncing off the curved walls of the potential. This bouncing will be highly random.

Naively, one would expect that on average the fields become confined in the region

$$\langle\phi^2\rangle = \langle\chi^2\rangle \sim \frac{v}{g}\,. \tag{D.1}$$

This result would coincide with our estimate for the amplitude of the oscillations of ϕ at the end of the stage of parametric resonance, cf. (3.21).

However, the situation is more complicated. As we are going to show, the fields spend most of the time not at $|\phi| \sim |\chi| \sim \sqrt{v/g}$, but exponentially far away from this region, moving along one of the flat directions of the potential.

To see this, note that because of the chaotic nature of the bouncing, it will occasionally happen that the fields enter the valley $\chi \ll \phi$ at a small angle to the flat direction, i.e. with velocities obeying $|\dot{\chi}| \ll \dot{\phi}$. Defining $|\dot{\chi}| \equiv \alpha v$, we are interested in the case that the angle α happens to be small.

Energy conservation implies that the amplitude of the oscillations of χ at the initial stage of this process is approximately $\alpha v/g\phi$. Because of the interaction term $\frac{g^2}{2}\phi^2\chi^2$, these oscillations act on the field ϕ with an average returning force $\sim \alpha^2 v^2/\phi$, which corresponds to the logarithmic potential $V(\phi) \sim \alpha^2 v^2 \log\phi$ [44]. Clearly, this potential will eventually pull the field ϕ back to the ESP $\phi = 0$. However, this happens at exponentially large ϕ: the field starts moving back only after its value approaches

$$\phi^*_{\rm class} \sim \sqrt{\frac{v}{g}}\, e^{c/\alpha}\,, \tag{D.2}$$

where $c = \mathcal{O}(1)$.

Once again, because the bouncing process is highly random, we do not expect that the probability to enter the valley at a small angle α is exponentially suppressed. This means that after bouncing back and forth near the point $\phi = \chi = 0$, the fields ϕ and χ eventually enter one of the valleys at a small angle, and subsequently spend a very long time there. In general, the fields will spend an exponentially long time at an exponentially large distance from the origin. Thus, the classical trapping mechanism, unlike the particle production mechanism described in our paper, does not lead to a permanent trapping of the fields in the vicinity of the point $\phi = \chi = 0$.

References

[1] L. Kofman, A.D. Linde and A.A. Starobinsky, *Towards the theory of reheating after inflation*, *Phys. Rev.* **D 56** (1997) 3258 [hep-ph/9704452].

[2] G.N. Felder, L. Kofman and A.D. Linde, *Instant preheating*, *Phys. Rev.* **D 59** (1999) 123523 [hep-ph/9812289].

[3] G.N. Felder, L. Kofman and A.D. Linde, *Inflation and preheating in no models*, *Phys. Rev.* **D 60** (1999) 103505 [hep-ph/9903350].

[4] D.J.H. Chung, E.W. Kolb, A. Riotto and I.I. Tkachev, *Probing planckian physics: resonant production of particles during inflation and features in the primordial power spectrum*, *Phys. Rev.* **D 62** (2000) 043508 [hep-ph/9910437].

[5] E. Silverstein and D. Tong, *Scalar speed limits and cosmology: acceleration from d-cceleration*, hep-th/0310221;
M. Alishahiha, E. Silverstein and D. Tong, in progress.

[6] E. Witten, *Bound states of strings and p-branes*, *Nucl. Phys.* **B 460** (1996) 335 [hep-th/9510135].

[7] N. Seiberg and E. Witten, *Electric-magnetic duality, monopole condensation and confinement in $N = 2$ supersymmetric Yang-Mills theory*, *Nucl. Phys.* **B 426** (1994) 19 [hep-th/9407087]; *Monopoles, duality and chiral symmetry breaking in $N = 2$ supersymmetric QCD*, *Nucl. Phys.* **B 431** (1994) 484 [hep-th/9408099];

K.A. Intriligator and N. Seiberg, *Lectures on supersymmetric gauge theories and electric-magnetic duality*, *Nucl. Phys.* **45BC** *(Proc. Suppl.)* (1996) 1 [hep-th/9509066].

[8] A. Strominger, *Massless black holes and conifolds in string theory*, *Nucl. Phys.* **B 451** (1995) 96 [hep-th/9504090].

[9] E. Witten, *String theory dynamics in various dimensions*, *Nucl. Phys.* **B 443** (1995) 85 [hep-th/9503124];
S. Katz, D.R. Morrison and M. Ronen Plesser, *Enhanced gauge symmetry in type-II string theory*, *Nucl. Phys.* **B 477** (1996) 105 [hep-th/9601108];
M. Bershadsky, K. Intriligator, S. Kachru, D.R. Morrison, V. Sadov, and C. Vafa, *Geometric singularities and enhanced gauge symmetries*, *Nucl. Phys.* **B 481** (1996) 215 [hep-th/9605200].

[10] E. Witten, *Small instantons in string theory*, *Nucl. Phys.* **B 460** (1996) 541 [hep-th/9511030].

[11] M. Dine, *Towards a solution of the moduli problems of string cosmology*, *Phys. Lett.* **B 482** (2000) 213 [hep-th/0002047];
M. Dine, Y. Nir and Y. Shadmi, *Enhanced symmetries and the ground state of string theory*, *Phys. Lett.* **B 438** (1998) 61 [hep-th/9806124].

[12] A. Berera and T.W. Kephart, *Ubiquitous inflaton in string-inspired models*, *Phys. Rev. Lett.* **83** (1999) 1084 [hep-ph/9904410].

[13] N.D. Birrell and P.C.W. Davies, *Quantum fields in curved space*, Cambridge University Press, Cambridge 1982.

[14] A.D. Linde, *Hybrid inflation*, *Phys. Rev.* **D 49** (1994) 748 [astro-ph/9307002].

[15] G.N. Felder, J. Garcia-Bellido, P.B. Greene, L. Kofman, A.D. Linde and I. Tkachev, *Dynamics of symmetry breaking and tachyonic preheating*, *Phys. Rev. Lett.* **87** (2001) 011601 [hep-ph/0012142].

[16] L. Kofman, A.D. Linde and A.A. Starobinsky, *Non-thermal phase transitions after inflation*, *Phys. Rev. Lett.* **76** (1996) 1011 [hep-th/9510119];
G.N. Felder, L. Kofman, A.D. Linde and I. Tkachev, *Inflation after preheating*, *J. High Energy Phys.* **08** (2000) 010 [hep-ph/0004024].

[17] G. Dvali and S. Kachru, *New old inflation*, hep-th/0309095.

[18] R. Easther, J. Khoury and K. Schalm, *Tuning locked inflation: supergravity versus phenomenology*, hep-th/0402218.

[19] L. Järv, T. Mohaupt and F. Saueressig, *Effective supergravity actions for flop transitions*, *J. High Energy Phys.* **12** (2003) 047 [hep-th/0310173]; *M-theory cosmologies from singular Calabi-Yau compactifications*, *JCAP* **02** (2004) 012 [hep-th/0310174].

[20] M. Brandle and A. Lukas, *Flop transitions in M-theory cosmology*, *Phys. Rev.* **D 68** (2003) 024030 [hep-th/0212263].

[21] T. Banks, W. Fischler and L. Motl, *Dualities versus singularities*, *J. High Energy Phys.* **01** (1999) 019 [hep-th/9811194];
L. Motl and T. Banks, *On the hyperbolic structure of moduli spaces with 16 SUSYs*, *J. High Energy Phys.* **05** (1999) 015 [hep-th/9904008].

JHEP05(2004)030

[22] M.R. Douglas, D. Kabat, P. Pouliot and S.H. Shenker, *D-branes and short distances in string theory*, *Nucl. Phys.* **B 485** (1997) 85 [hep-th/9608024].

[23] C. Bachas, *D-brane dynamics*, *Phys. Lett.* **B 374** (1996) 37 [hep-th/9511043].

[24] J.H. Horne and G.W. Moore, *Chaotic coupling constants*, *Nucl. Phys.* **B 432** (1994) 109 [hep-th/9403058].

[25] T. Banks, M. Berkooz, S.H. Shenker, G.W. Moore and P.J. Steinhardt, *Modular cosmology*, *Phys. Rev.* **D 52** (1995) 3548 [hep-th/9503114].

[26] S.Y. Khlebnikov and I.I. Tkachev, *Classical decay of inflaton*, *Phys. Rev. Lett.* **77** (1996) 219 [hep-ph/9603378].

[27] G.N. Felder and I. Tkachev, *LATTICEEASY: a program for lattice simulations of scalar fields in an expanding universe*, hep-ph/0011159.

[28] L. Randall and R. Sundrum, *A large mass hierarchy from a small extra dimension*, *Phys. Rev. Lett.* **83** (1999) 3370 [hep-ph/9905221].

[29] G.N. Felder and L. Kofman, *The development of equilibrium after preheating*, *Phys. Rev.* **D 63** (2001) 103503 [hep-ph/0011160].

[30] G.N. Felder, A.V. Frolov, L. Kofman and A.V. Linde, *Cosmology with negative potentials*, *Phys. Rev.* **D 66** (2002) 023507 [hep-th/0202017].

[31] A. Strominger, *Open string creation by S-branes*, hep-th/0209090.

[32] A. Maloney, A. Strominger and X. Yin, *S-brane thermodynamics*, *J. High Energy Phys.* **10** (2003) 048 [hep-th/0302146].

[33] S.S. Gubser, *String production at the level of effective field theory*, hep-th/0305099; *String creation and cosmology*, hep-th/0312321;
J.J. Friess, S.S. Gubser and I. Mitra, *String creation in cosmologies with a varying dilaton*, hep-th/0402156.

[34] A.D. Linde, *Inflation can break symmetry in SUSY*, *Phys. Lett.* **B 131** (1983) 330.

[35] A.D. Linde, *Inflation and quantum cosmology*, in *300 years of gravitation*, S.W. Hawking and W. Israel eds., Cambridge University Press, Cambridge 1987.

[36] A.D. Linde, D.A. Linde and A. Mezhlumian, *From the big bang theory to the theory of a stationary universe*, *Phys. Rev.* **D 49** (1994) 1783 [gr-qc/9306035];
J. Garcia-Bellido, A.D. Linde and D.A. Linde, *Fluctuations of the gravitational constant in the inflationary Brans-Dicke cosmology*, *Phys. Rev.* **D 50** (1994) 730 [astro-ph/9312039];
J. Garriga and A. Vilenkin, *A prescription for probabilities in eternal inflation*, *Phys. Rev.* **D 64** (2001) 023507 [gr-qc/0102090];
A. Linde, *Inflation, quantum cosmology and the anthropic principle*, in *Science and ultimate reality: from quantum to cosmos*, J.D. Barrow, P.C.W. Davies and C.L. Harper eds., Cambridge University Press, Cambridge 2003 [hep-th/0211048];
J. Garriga, A. Linde and A. Vilenkin, *Dark energy equation of state and anthropic selection*, *Phys. Rev.* **D 69** (2004) 063521 [hep-th/0310034].

[37] R. Bousso and J. Polchinski, *Quantization of four-form fluxes and dynamical neutralization of the cosmological constant*, *J. High Energy Phys.* **06** (2000) 006 [hep-th/0004134].

[38] A. Maloney, E. Silverstein and A. Strominger, *de Sitter space in noncritical string theory*, hep-th/0205316;
E. Silverstein, *(A)dS backgrounds from asymmetric orientifolds*, hep-th/0106209.

[39] S. Kachru, R. Kallosh, A. Linde and S.P. Trivedi, *de Sitter vacua in string theory*, *Phys. Rev.* **D 68** (2003) 046005 [hep-th/0301240].

[40] M.R. Douglas, *The statistics of string/M theory vacua*, *J. High Energy Phys.* **05** (2003) 046 [hep-th/0303194];
S. Ashok and M.R. Douglas, *Counting flux vacua*, *J. High Energy Phys.* **01** (2004) 060 [hep-th/0307049].

[41] B.S. Acharya, *A moduli fixing mechanism in M-theory*, hep-th/0212294.

[42] S. Gukov, S. Kachru, X. Liu and L. McAllister, *Heterotic moduli stabilization with fractional Chern-Simons invariants*, *Phys. Rev.* **D 69** (2004) 086008 [hep-th/0310159].

[43] R. Fardon, A.E. Nelson and N. Weiner, *Dark energy from mass varying neutrinos*, astro-ph/0309800.

[44] R. Helling, *Beyond eikonal scattering in M(atrix)-theory*, hep-th/0009134.

JHEP05(2004)030